Investigation in modelling piping erosion with a coupled « lattice Boltzmann – discrete element » numerical method

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Internal erosion in soils
A fully solid-fluid coupled phenomena
The solid-fluid coupled numerical method

- Description of the solid phase at the particle scale
- Description of the fluid dynamic in the inter-particle space

**Solid phase: Discrete Element Method**
DEM, *Yade Software*

- Contact stiffnesses
- Contact friction angle
- Contact adhesion

**Fluid phase**
Lattice Boltzmann Method (LBM)

- Fluid viscosity
- Position of each solid particle explicitly described

Particle positions and velocities
hydro-dynamic forces

No assumption on fluid/solid interactions:
permeability, drag forces, etc... result from the coupling.
The solid-fluid coupled numerical method
The discrete element method (solid phase)

- Solid phase explicitly described by an assembly of particles

- Computation step:
  
  Computation of grain positions
  by integration of Newton's law for each grain.
  
  \[ \ddot{x}_i = \frac{F_i}{m} \]
  \[ \ddot{\omega}_i = \frac{M_i}{J} \]

  Computation of Contact forces
  Thanks to the interaction contact law defined for each contact.

  Computation of resulting force and torque acting on each grain

- Explicit integration scheme

- Open source software YADE (Wiki: yade-dem.org)
The solid-fluid coupled numerical method
Lattice Boltzmann method (fluid phase)

⇒ Based on the probability density or distribution function $f(\vec{x}, t)$
representing the probability of finding a molecule (or particle) around position $\vec{x}$ at time $t$
with a given momentum.

⇒ The BGK (Bhatnagar-Gross-Krook, 1954) collision operator
describes the time and spatial evolution of a distribution function (i.e. of momentum):

$$f(\vec{x}, t^+) = f(\vec{x}, t) - \frac{1}{\tau} \left[ f(\vec{x}, t) - f_{eq}(\vec{x}, t) \right]$$

with $\tau = 3 \nu dt / h^2 + 1 / 2$

⇒ Transfer of momentum from the solid particles to the fluid at solid boundaries
distribution functions affected by a term involving the solid boundary velocity $\vec{V}_b$

$$f_{-\sigma i}(\vec{x}_{FB}, t + dt) = f_{\sigma i}(\vec{x}_{FB}, t^+) - 2\alpha_i \vec{V}_b \cdot \vec{e}_i$$

⇒ Force applied by the fluid on the solid
results from the time derivation of the momentum exchange at solid boundaries

$$\vec{F}_\sigma(\vec{x}, t + \frac{1}{2} dt) = 2 \frac{\Omega}{dt} \left[ f_{\sigma i}(\vec{x}, t^+) - \alpha_i \vec{V}_b \cdot \vec{e}_i \right] \vec{e}_{\sigma i}$$

boundary link $\sigma$
The solid-fluid coupled numerical method
The discrete element method (solid phase)

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\[
\begin{align*}
\dot{x}_i &= F_i / m \\
\dot{\omega}_i &= M_i / J
\end{align*}
\]

with:

\[
\begin{align*}
\vec{F} &= \vec{F}^C + \vec{F}^H \\
\vec{M} &= \vec{M}^C + \vec{M}^H
\end{align*}
\]
Application to piping erosion
Model description

⇒ Simplified 2D Hole Erosion Test (HET):
- Cohesive frictional granular assembly:
  \[ \phi_c = 20^\circ \quad C = -C_n = C_s \]
- Initial hole drilled in the granular assembly,
- Water flow under constant pressure gradient: \( \Delta P = P_1 - P_2 \).

⇒ Brittle cohesive inter-particle contacts:

![Diagram showing normal contact force (F_n) vs. shear contact force (F_s) with cohesion broken at C_n = C_s = 0.](image)

Lominé et al. IJNAMG, 2011
Application to piping erosion
Characterisation of erodability

⇒ Classical interpretation with respect to the hydraulic shear stress $\tau$:

$$\dot{\epsilon} = k_d \left( \tau - \tau_c \right) \text{ if } \tau > \tau_c$$

($\tau_c$ : critical shear stress
$k_d$ : erosion coefficient.

⇒ Hydraulic shear stress computed along the hole border:

$$\tau = \nu \rho_0 \frac{dV_x}{dy}$$

$Lominé et al. IJNAMG, 2011$
Application to piping erosion
Influence of inter-particle cohesion

⇒ 7 values of cohesion tested, each one for 6 to 10 different values of $\Delta P$:

$$C/d = 0.152; 0.177; 0.253; 0.506; 1.27; 2.53; 12.7 \text{ N/m}$$

→ $\tau_c$ directly affected by cohesion for cohesion values high enough ($C/d > 0.506 \text{ N/m}$).

→ $k_d$ seems independent of the cohesion.
Application to piping erosion
Energetic interpretation

Energy supplied to the fluid is almost completely dissipated by viscosity:

\[ Q \Delta P \approx \int_{V} \vec{\sigma}' \cdot \vec{D} \, dV \]

Shear stress \( \propto \vec{D} \)
Viscous fluid power \( \propto \vec{D} : \vec{D} \)

\( \Rightarrow \) Is the erosion rate linearly related to the square root of the viscous fluid power?
Conclusions

- The coupled discrete element – lattice Boltzmann method can be a versatile numerical method to improve the understanding of soil erosion phenomena complementary to experiments.

- Estimation of energy dissipated by the fluid flow may be easier than the determination of the fluid shear stress an could help in the evaluation of internal soil erosion hazards.