Institut de Recherche en Génie Civil et Mécanique
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Investigation in modelling piping erosion with a coupled « lattice Boltzmann – discrete element » numerical method

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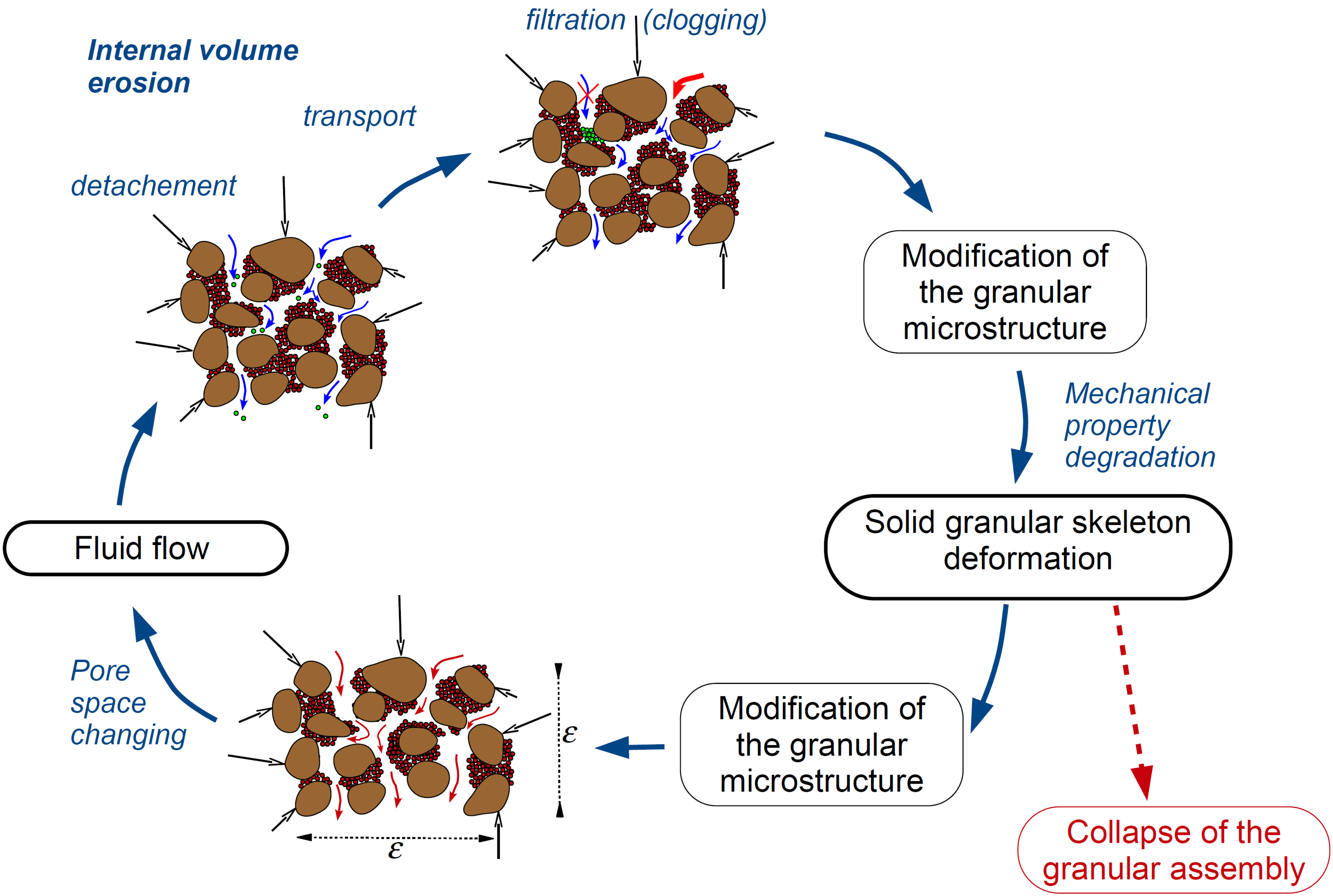
Internal erosion in soils

A fully solid-fluid coupled phenomena



The solid-fluid coupled numerical method

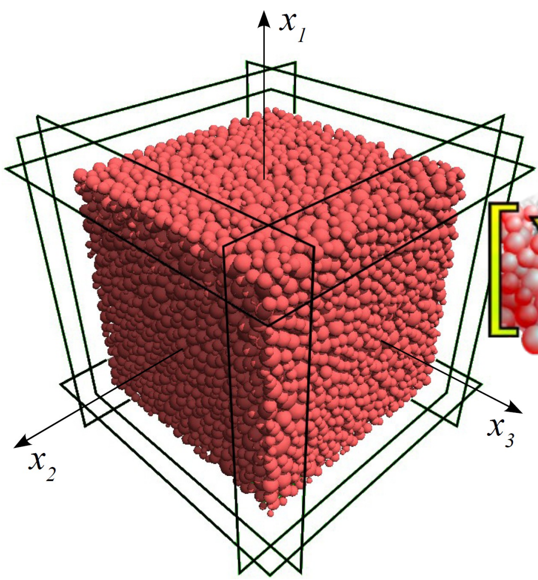
- **Description of the solid phase at the particle scale**
- **Description of the fluid dynamic in the inter-particle space**

Solid phase: Discrete Element Method DEM, *Yade Software*

- Contact stiffnesses
- Contact friction angle
- Contact adhesion

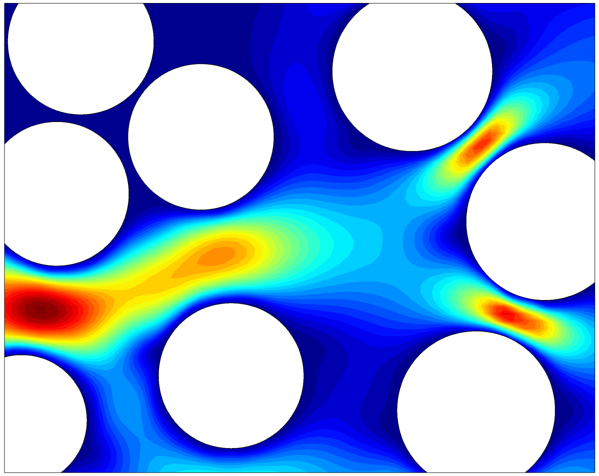
Fluid phase Lattice Boltzmann Method (LBM)

- Fluid viscosity
- position of each solid particle explicitely described



Particle positions and velocities

hydro-dynamic forces

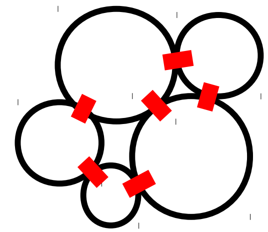


No assumption on fluid/solid interactions: permeability, drag forces, etc... result from the coupling.

The solid-fluid coupled numerical method

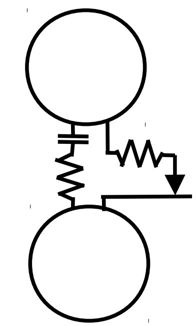
The discrete element method (solid phase)

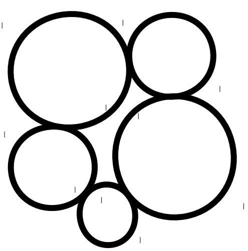
- Solid phase explicitly described by an assembly of particles



- Computation step:

Update of contact list





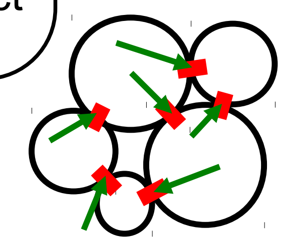
Computation of grain positions

by integration of Newton's law for each grain.

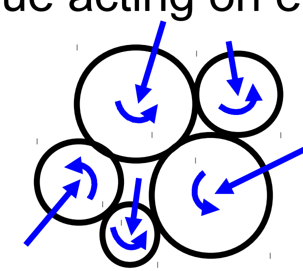
$$\ddot{x}_i = F_i / m$$
$$\ddot{\omega}_i = M_i / J$$

Computation of Contact forces

Thanks to the interaction contact law defined for each contact.



Computation of resulting force and torque acting on each grain



- Explicit integration scheme

- Open source software YADE (Wiki: yade-dem.org)

The solid-fluid coupled numerical method

Lattice Boltzmann method (fluid phase)

⇒ **Based on the probability density or distribution function** $f(\vec{x},t)$

representing the probability of finding a molecule (or particle) around position $\vec{x}$ at time t with a given **momentum**.

⇒ **The BGK (Bhatnagar-Gross-Krook, 1954) collision operator**

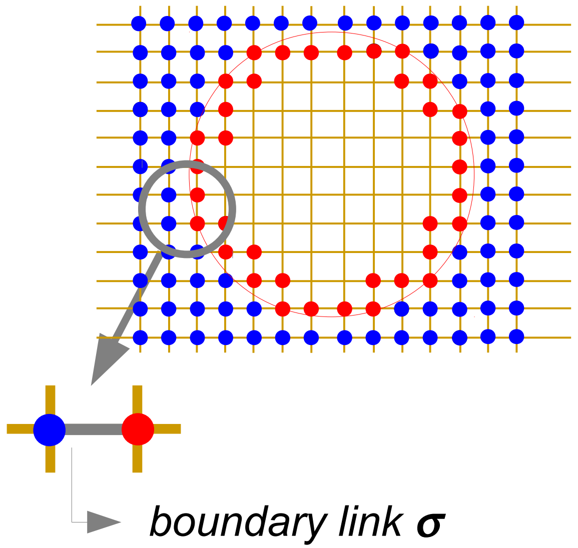
describes the time and spatial evolution of a distribution function (i.e. of momentum):

$$f(\vec{x},t^{+})=f(\vec{x},t)-\frac{1}{\tau}\left[f(\vec{x},t)-f^{eq}(\vec{x},t)\right] \qquad \text{with } \tau=3\nu\, dt/h^{2}+1/2$$

⇒ **Transfer of momentum from the solid particles to the fluid at solid boundaries**

distribution functions affected by a terms involving the solid boundary velocity $\vec{V}_b$

$$f_{-\sigma i}(\vec{x}_{FB},t+\mathrm{d}t)=f_{\sigma i}(\vec{x}_{FB},t^{+})-2\alpha_i\vec{V_b}.\vec{e_i}$$



⇒ **Force applied by the fluid on the solid**

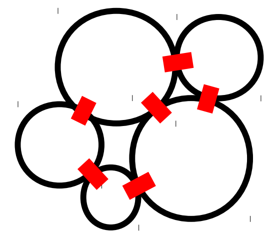
results from the time derivation of the momentum exchange at solid boundaries

$$\vec{F}_\sigma(\vec{x},t+\frac{1}{2}\mathrm{d}t)=2\frac{\Omega}{\mathrm{d}t}\left[f_{\sigma i}(\vec{x},t^{+})-\alpha_i\vec{V_b}.\vec{e_i}\right]\vec{e}_{\sigma i}$$

The solid-fluid coupled numerical method

The discrete element method (solid phase)

- Solid phase explicitly described by an assembly of particles
- Computation step:



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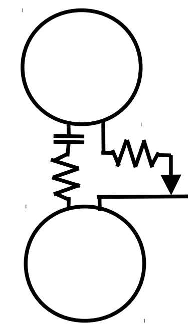

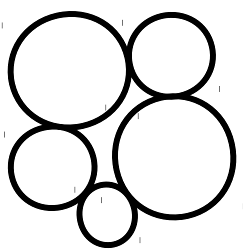

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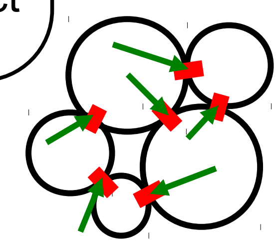
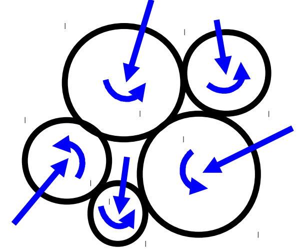


$$\ddot{x}_i = F_i / m$$
$$\ddot{\omega}_i = M_i / J$$

Computation of resulting force and torque acting on each grain

with:
$$\vec{F} = \vec{F}^C + \vec{F}^H$$
$$\vec{M} = \vec{M}^C + \vec{M}^H$$



Application to piping erosion
Model description

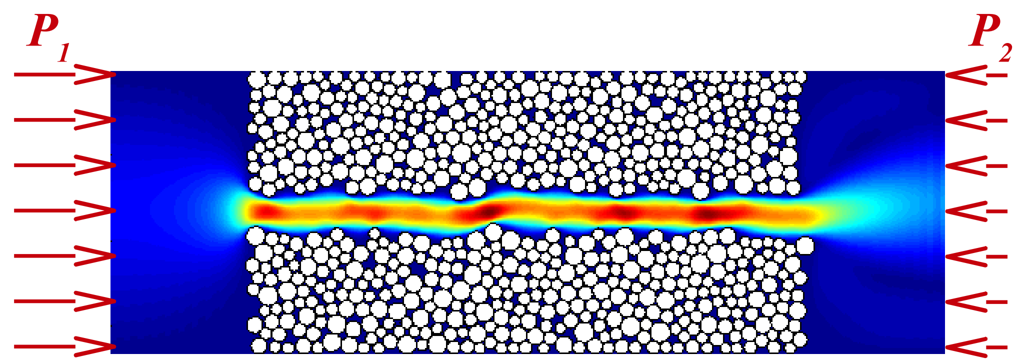
⇒ Simplified 2D Hole Erosion Test (HET):

Lominé et al. IJNAMG, 2011

- Cohesive frictional granular assembly:

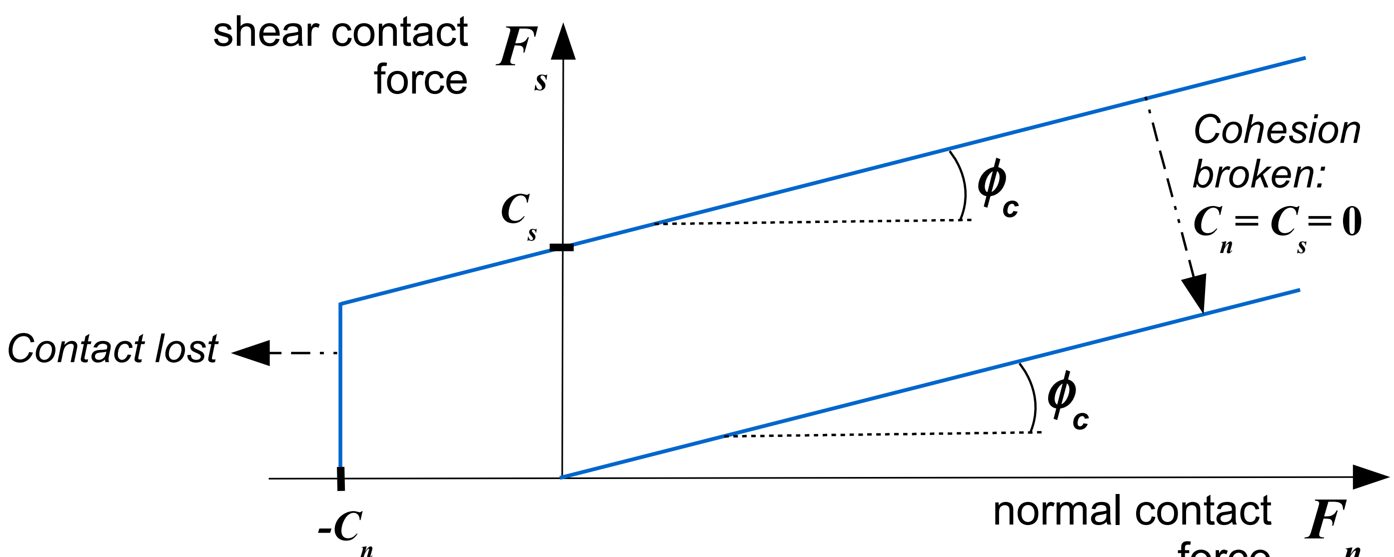
$$\phi_C = 20° \qquad C = -C_n = C_s$$

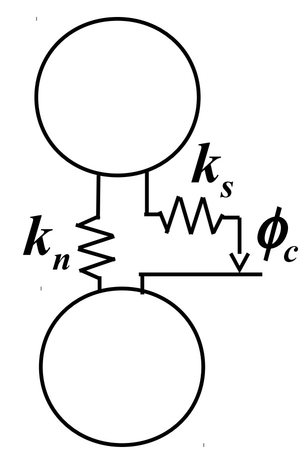
- Initial hole drilled in the granular assembly,
- Water flow under constant pressure gradient: $\Delta P = P_1 - P_2$.



800 solid particles; fluid lattice of 335 000 nodes

⇒ Brittle cohesive inter-particle contacts:





Application to piping erosion

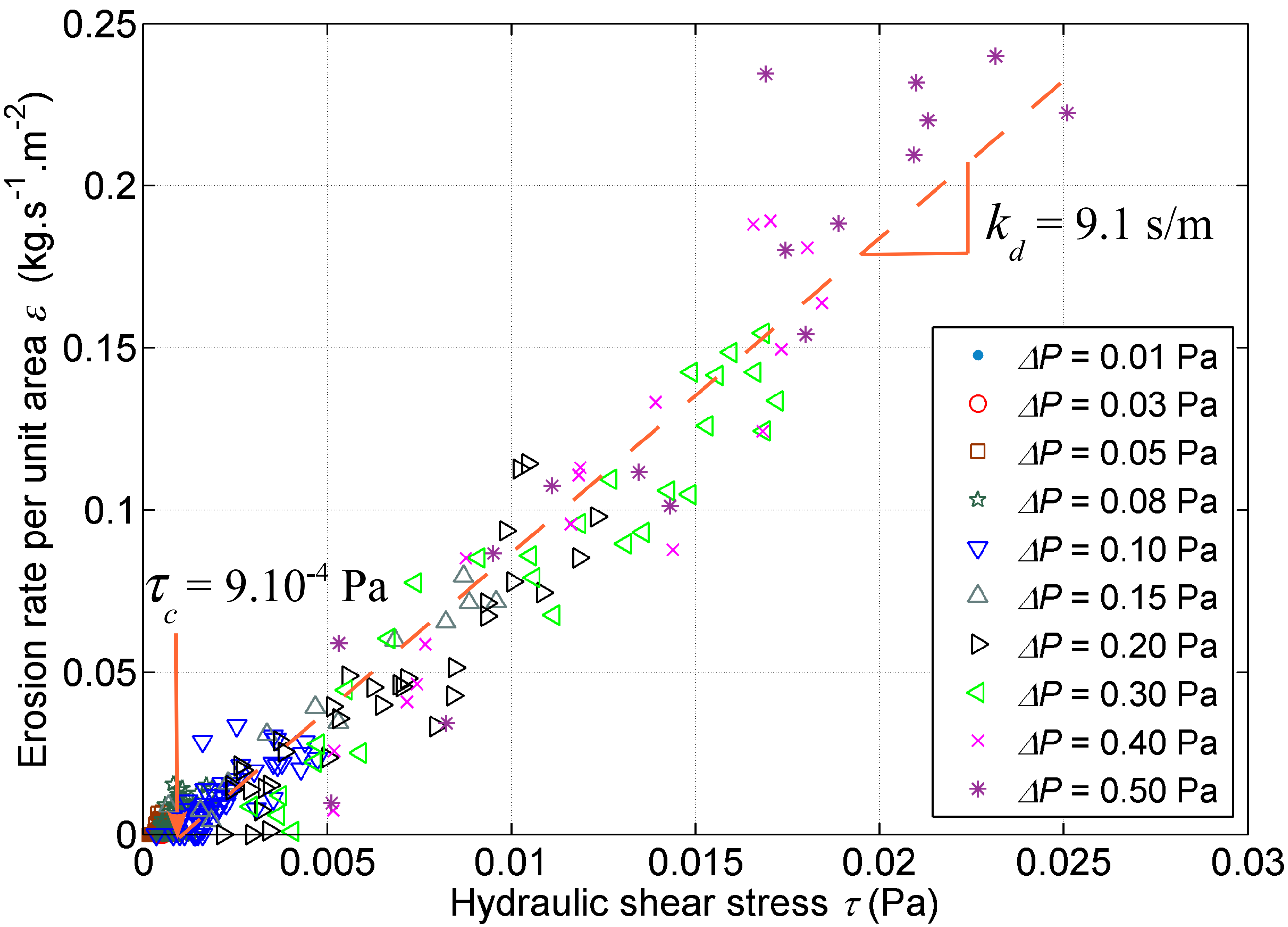
Characterisation of erodability

⇒ Classical interpretation with respect to the hydraulic shear stress τ :

$$\dot{\epsilon} = k_d\,(\tau - \tau_c) \text{ if } \tau > \tau_c$$ (Shields 1936, Wan & Fell 2002)

τ_c : critical shear stress

k_d : erosion coefficient.



→ Hydraulic shear stress computed along the hole border:

$$\tau = \nu \rho_0 \frac{\mathrm{d}V_x}{\mathrm{d}y}$$

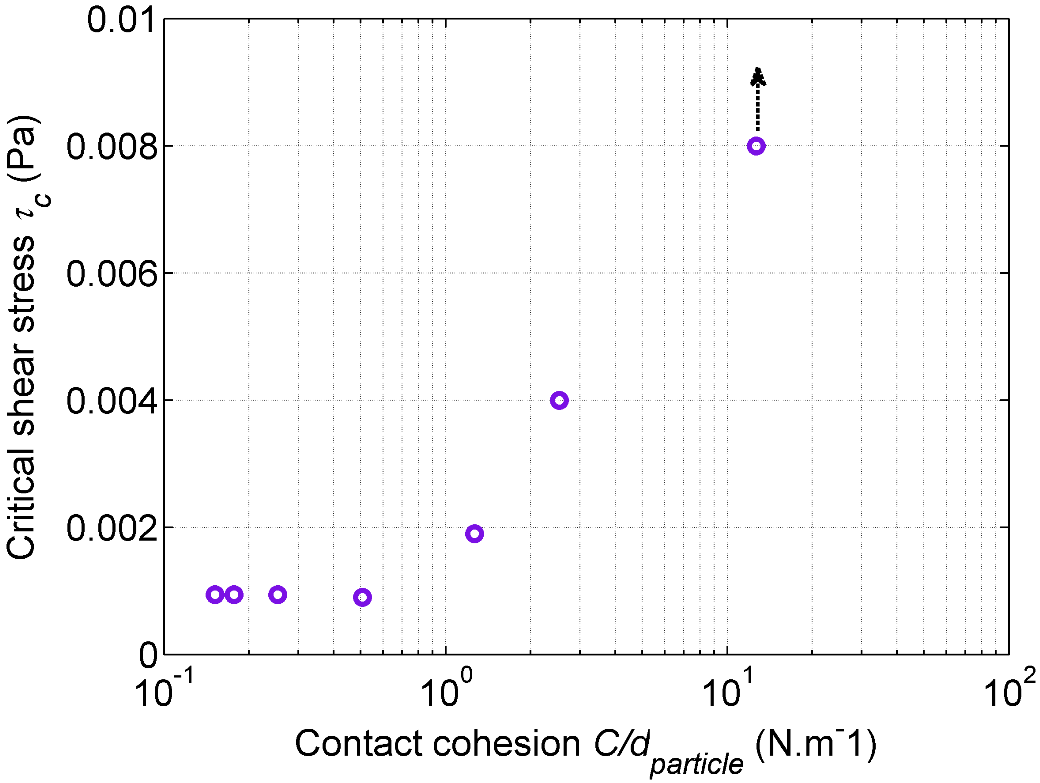
Lominé et al. IJNAMG, 2011

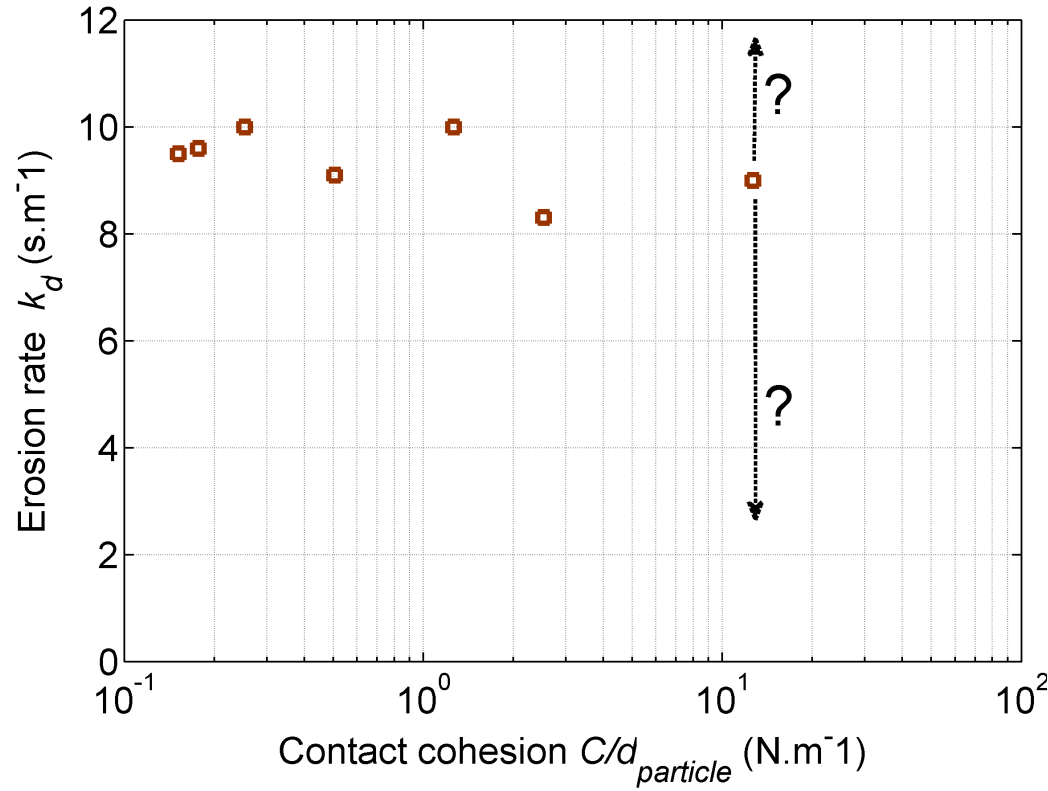
Application to piping erosion

Influence of inter-particle cohesion

⇒ 7 values of cohesion tested, each one for 6 to 10 different values of ΔP :

C/d = 0.152; 0.177; 0.253; 0.506; 1.27; 2.53; 12.7 N/m





→ τ_c directly affected by cohesion for cohesion values high enough (C/d > 0.506 N/m).

→ k_d seems independent of the cohesion.

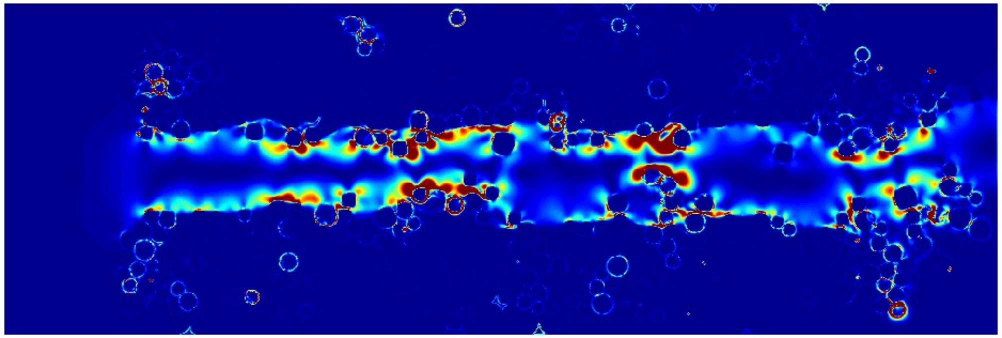
Application to piping erosion

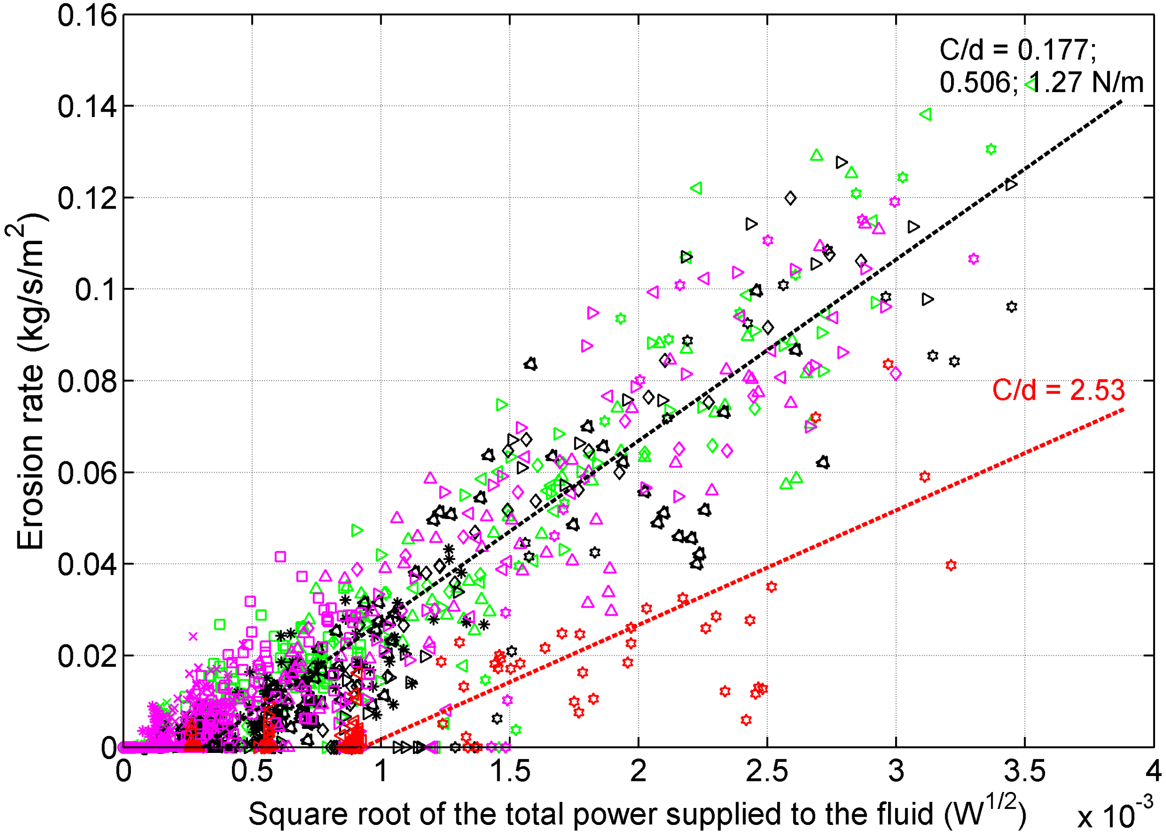
Energetic interpretation

Energy supplied to the fluid is almost completely dissipated by viscosity:

$$Q \Delta P \simeq \int_V \bar{\bar{\sigma}}' : \bar{\bar{D}} \, dV$$

Power dissipated by viscosity $\bar{\bar{\sigma}}' : \bar{\bar{D}}$





Shear stress $\propto \bar{\bar{D}}$

Viscous fluid power $\propto \bar{\bar{D}} : \bar{\bar{D}}$

⇒ Is the erosion rate linearly related to the square root of the viscous fluid power?

Conclusions

- The coupled discrete element – lattice Boltzmann method can be a versatile numerical method to improve the understanding of soil erosion phenomena complementary to experiments.

- Estimation of energy dissipated by the fluid flow may be easier than the determination of the fluid shear stress an could help in the evaluation of internal soil erosion hazards