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A continuum model for infiltration problems

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Outline

- Motivation
- Continuum modeling
- Constitutive equation for mass exchange
- Numerical example
- Conclusion

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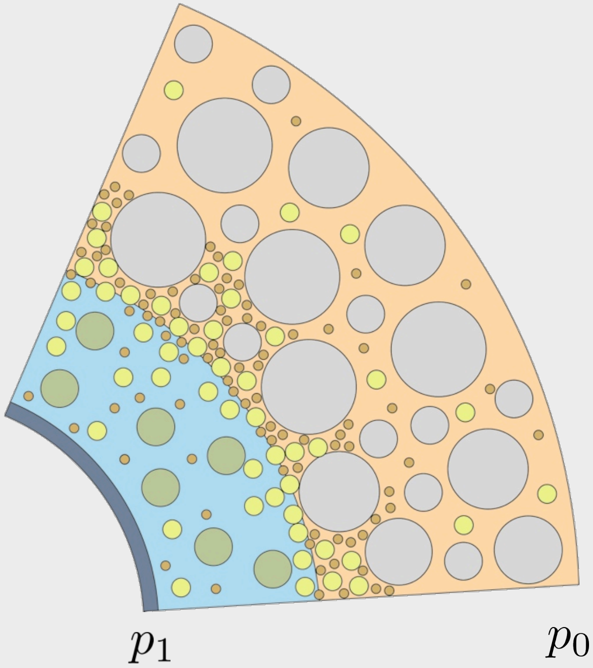
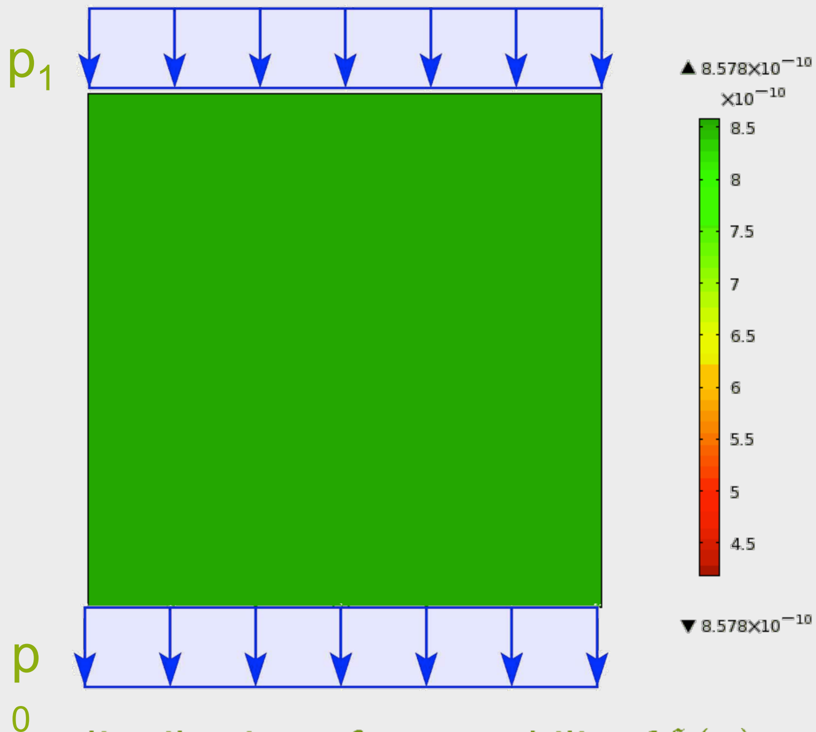
Motivation - Infiltration

- Focus on the percolation of a porous media with a suspension
- Infiltration (phase transition of single constituents) varies hydraulic properties of the considered domain in space and time
- Capturing rearrangement effects, i.e. formation of an internal-, external filter cake, clogging
- Four-phase continuum model for describing infiltration problems
- Main application: backfilling of the annular gap with grouting mortar in the field of mechanized tunneling

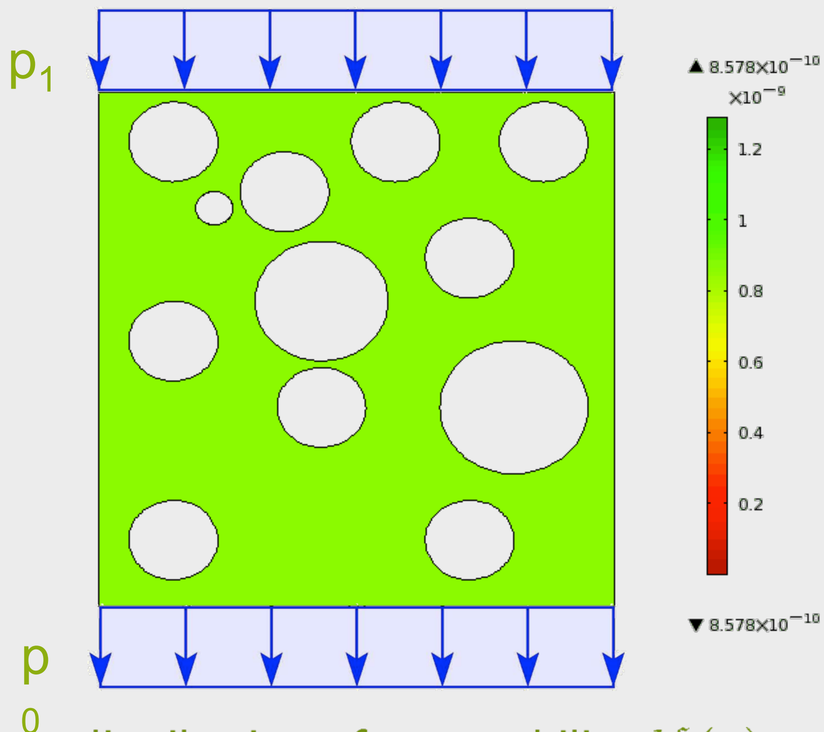


$p_1 > p_0$

Motivation - Infiltration



distribution of permeability $k^s(x)$



distribution of permeability $k^s(x)$

- Non permeable heterogeneities initialize significant change in the permeability of the volume
- ➢ Formation of a filter cake

Hydraulic Multiscale Model

Framework

- Superimposed continua (Truesdell [1966])
- Mixture theory extended by concepts of volume fractions (Bowen [1980,1982], de Boer [2000], Ehlers [2002], Steeb [2008], Coussy [2005])

Model assumptions

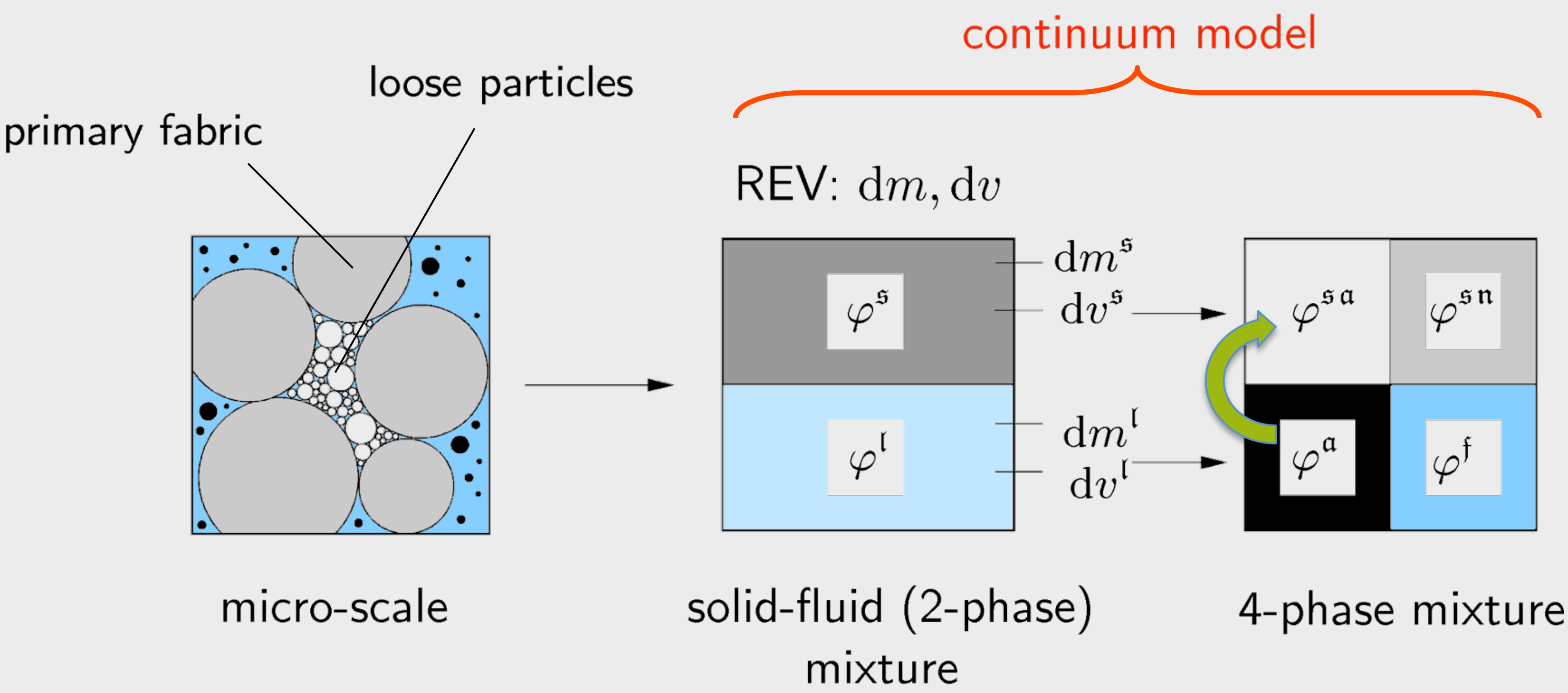
- Fully saturated mixture

$$n^\alpha := \frac{\mathrm{d}v^\alpha}{\mathrm{d}v}, \qquad \sum_\alpha n^\alpha = 1, \qquad \forall\, \alpha \in \{\mathfrak{sn}, \mathfrak{sa}, \mathfrak{f}, \mathfrak{a}\}$$

- Material incompressible constituents

$$\rho^{\alpha R} = \frac{\mathrm{d}m^\alpha}{\mathrm{d}v^\alpha} = \mathrm{const} \qquad \text{and} \qquad \rho^\alpha = \frac{\mathrm{d}m^\alpha}{\mathrm{d}v} = n^\alpha\, \rho^{\alpha R}$$

Hydraulic Multiscale Model



Reformulation in $\{\phi, c, a\}$

$$n^{\alpha} = \{n^{\mathfrak{s}}, n^{\mathfrak{l}}\} \qquad n^{\mathfrak{l}} = n^{\mathfrak{a}} + n^{\mathfrak{f}}$$

$$n^{\mathfrak{s}} = n^{\mathfrak{sa}} + n^{\mathfrak{sn}}$$

$$n^{\alpha} = \{n^{\mathfrak{sa}}, n^{\mathfrak{sn}}, n^{\mathfrak{a}}, n^{\mathfrak{f}}\}$$

$$c = \frac{n^{\mathfrak{a}}}{\phi}, \qquad \phi = n^{\mathfrak{f}} + n^{\mathfrak{a}}$$

$$a = \frac{n^{\mathfrak{sa}}}{n^{\mathfrak{s}}}, \quad n^{\mathfrak{s}} = n^{\mathfrak{sn}} + n^{\mathfrak{sa}} = 1 - \phi$$

Hydraulic Multiscale Model

Darcy relation

$$\mathbf{q} = -\frac{k^{\mathfrak{s}}(\phi)}{\eta^{\mathfrak{l}R}(c)} \operatorname{grad} p$$

Kozeny-Carman equation

$$k^{\mathfrak{s}}(\phi) = k_0^{\mathfrak{s}} \left[\frac{\phi^3}{(1-\phi)^2}\right] \left[\frac{(1-\phi_0)^2}{\phi_0^3}\right]$$

$$k_0^{\mathfrak{s}} = \frac{1}{C_1} \frac{\phi_0^3}{(1-\phi_0)^2} D_{eq}^2$$

Amount of attached fines

$$a = a(\phi) = \frac{a_0(1-\phi_0) + \phi_0 - \phi}{(1-\phi)}$$

Viscosity of a dilute suspension

(Einstein [1906])

$$\eta^{\mathfrak{l}R}(c) = \eta^{\mathfrak{f}R}\left(1 + \frac{5}{2}c\right)$$

IBVP for Infiltration

$$\operatorname{div}\left[\frac{k^{\mathfrak{s}}(\phi)}{\eta^{\mathfrak{f}}(c)} \operatorname{grad} p\right] = 0$$

$$\partial_t(c\,\phi) + \operatorname{div}\left[c\,\frac{k^{\mathfrak{s}}(\phi)}{\eta^{\mathfrak{f}}(c)} \operatorname{grad} p\right] = \hat{n}^{\mathfrak{a}}$$

$$\partial_t \phi = \hat{n}^{\mathfrak{a}}$$

IC and BC

$$q = \mathbf{q}\cdot\mathbf{n} = \bar{q}, \quad p = \bar{p} \wedge c = \bar{c}$$

$$\phi = \phi_0 \wedge c = c_0$$

Constitutive equation for mass exchange

- Rigid skeleton (only hydraulics)

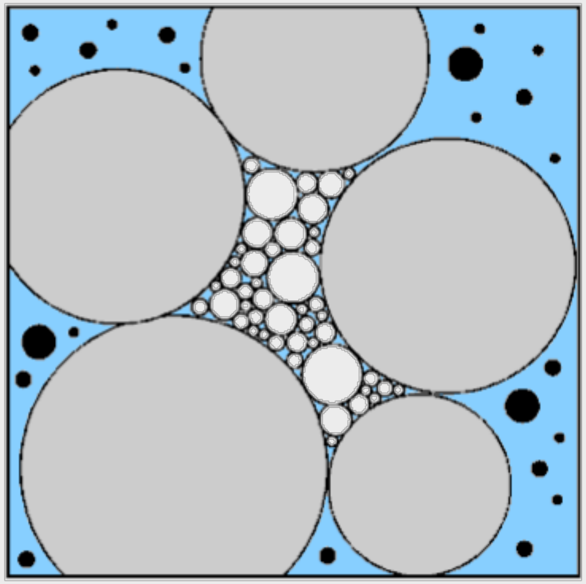
  $\hat{n}^{\mathfrak{a}} = -\hat{n}^{\mathfrak{s}\mathfrak{a}} =: -\hat{n}^{\mathfrak{s}}$

- Thermodynamically consistent mass exchange

  $\hat{n}^{\mathfrak{a}} \propto |\mathbf{q}|$

- Production term

  $\hat{n}^{\mathfrak{a}} = -\beta\, c\, |\mathbf{q}|$



β : material parameter

c : concentration of fluidized fines

$\mathbf{q}$: filter velocity

Characterization of the pore structure

- **Evaluation of the mass exchange term**

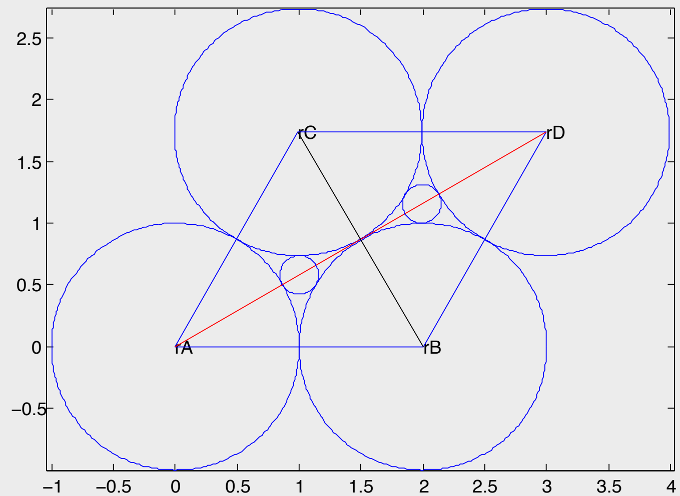
$$\beta := \kappa\phi$$

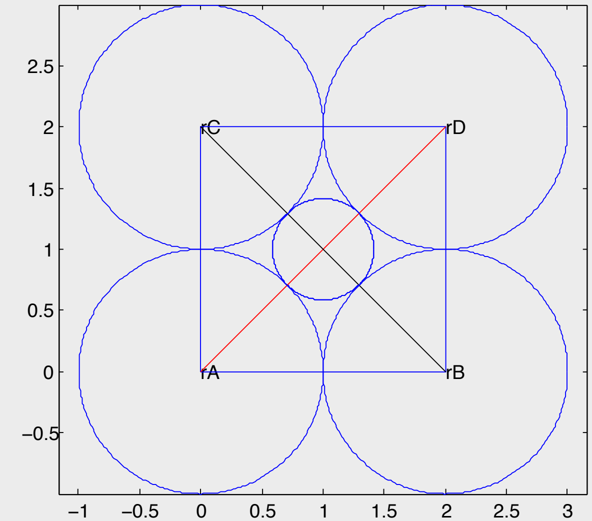
β : most general expression

κ : infiltrated fraction in one time step

$$0 \leq \kappa \leq c$$

- Determination of CSD in each time step and material point necessary





Determination of the CSD

- Starting with the GSD, the CSD can be determined by geometrical considerations

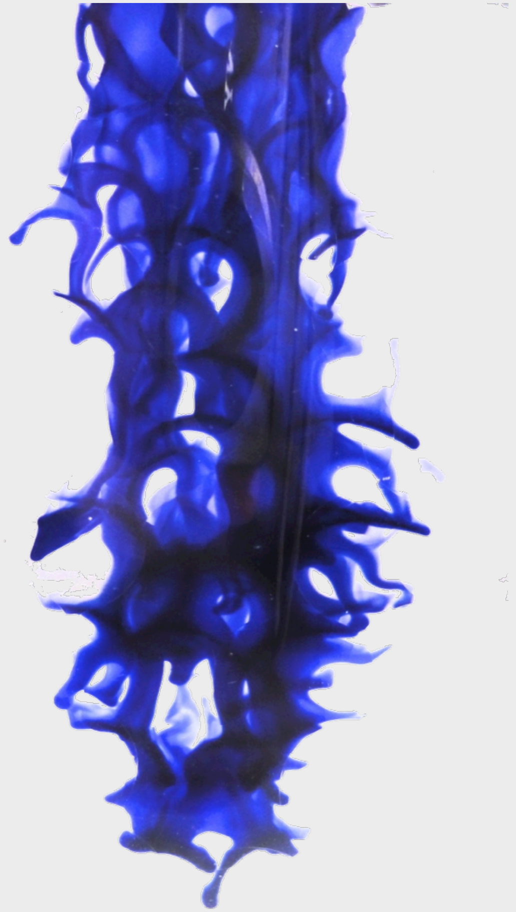
- Comparing the probability of occurrence of a fluidized particle together with a certain pore configuration, κ can be determined

- Assumption for critical ratio D/d is necessary

  - Validation by experiments

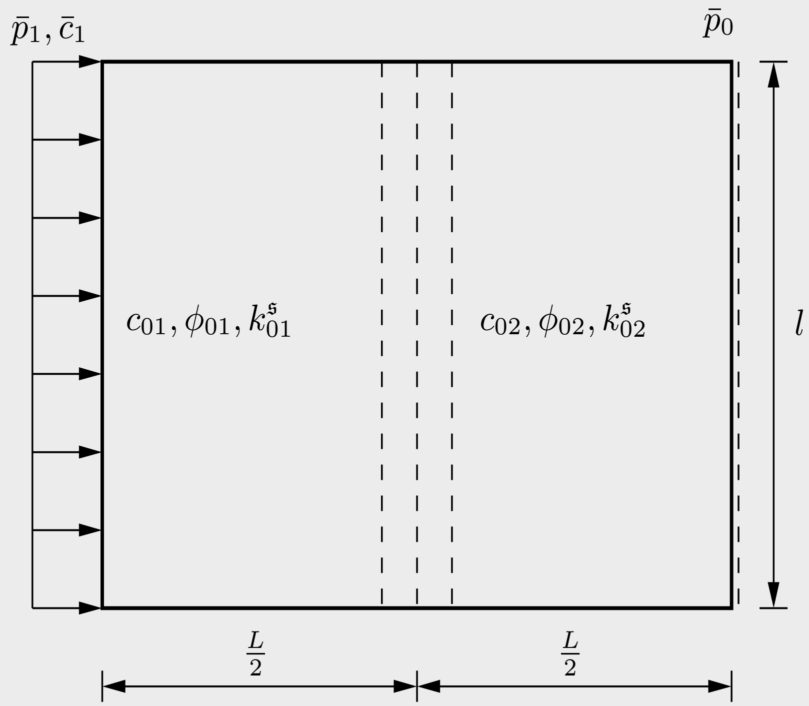
- For non uniform packing a κ parameter for each fraction has to be evaluated

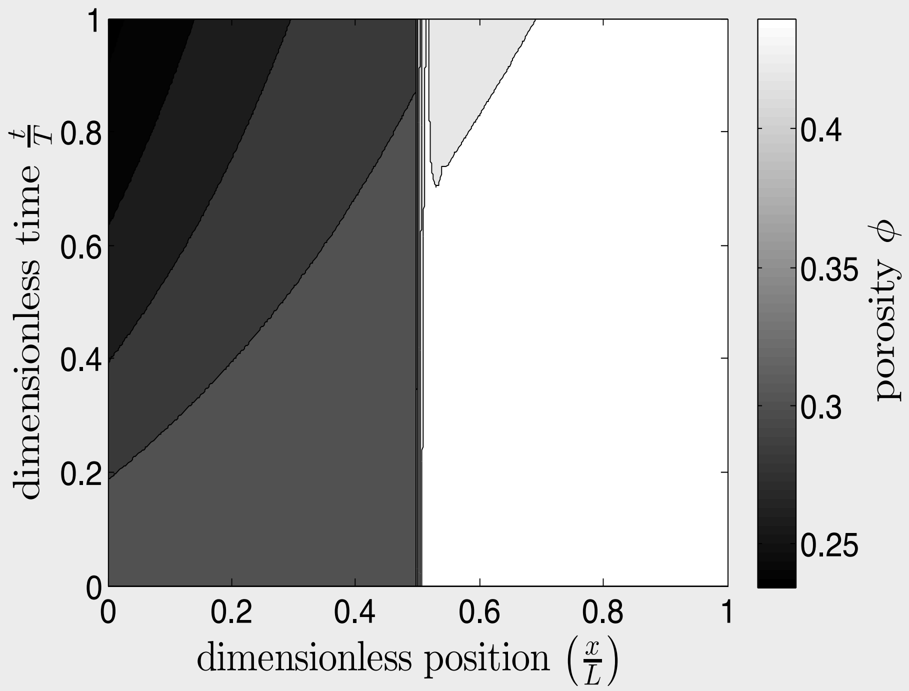
  E.g. Indraratna, B. & Radampola [2002], Witt, K.J. [1986]



Visualization of preferential flow paths and pore space in a porous media

Evolution of porosity in a 2-layer domain





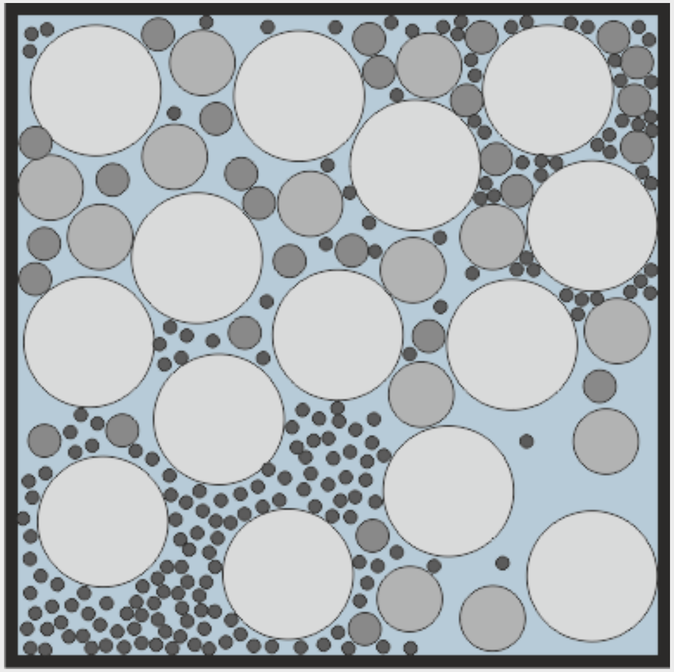
- Continuous infiltration process in the left part

cf. Schaufler et al. [2012]

- No significant change in porosity in the right part
  - Transport of the suspension through the pore space

Conclusion

- Formulation set of coupled/nonlinear PDE's describing infiltration processes
    - Implementation with coupled finite elements
- IBVP was extended by a thermodynamically consistent production term
    - Contains one material parameter
- Strong interaction between micro- and macro scale
    - Consideration of GSD during calculation



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Literature

thank you!

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