ELASTIC INTERPRETATION FOR SEDIMENT SHEET FLOWS

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Abstract: For the condition of high shear stresses, sediment particles can be transported in the form of sheet flows. Such collective motion of bed particles is described in this paper using a simple elastic model. The preliminary result shows that sediment transport rate for sheet flows is independent of sediment particle size, which generally agrees with previous observations. For comparison, it is also demonstrated that the effect of sediment size is not significant for sediment transport over a flat bed in laminar flows.

1. Introduction

For sediment beds subjected to sufficiently high shear stress, bedforms such as ripples and dunes may disappear, being followed by an intensively-moving layer of bed-load particles. The flow including such a layer is called sheet flow. Such flows are often associated with high transport rates, which present in many natural environments including rivers in flood, tidal estuaries, and beaches undergoing storm-wave attack, as well as in industrial applications such as sludge treatment and food stuff processing.

Sheet flows have been observed earlier by Wilson (1966), and recently by Sumer et al. (1995), Jenkins and Hanes (1998), and Nielsen et al. (2002), among others. Wilson (1966) conducted high-shear flow experiments with a pressurized pipe system rather than an open channel. His measurements have resulted in a significant improvement in the computation of bed load transport for high-shear conditions. Jenkins and Hanes (1998) approached the sheet flow problem numerically by applying kinetic theory for collisional grains. In addition, sediment transport induced by oscillatory sheet flows has been observed by Asano (1995) and Nielson et al. (2002). Other relevant studies are due to Wilson (1987, 1989), Wilson and Pugh (1988), Abrahams (2003), and Cheng (2003).

Since the sheet flow involves collective particle motion, its mechanism may not be necessarily explored by examining behaviors of individual particles. For example, the collective performance of particles has been examined by Ugawa and Sana (2002) for investigating standing waves composed of vertically oscillating particles. They reported that their experimental observations could be described reasonably using the theory of buckling developed for elastic materials. In this study, an elastic model is proposed for formulating sediment transport for the sheet flow condition.

2. Elastic Model

Consider a flat sediment bed subjected to a flow-induced shear stress, τ_b. As shown in Fig. 1, a vertical column is selected. Its height is the same as the thickness of the sediment sheet flow, δ. The dimension of the cross-section of the column is denoted as b. Here, we consider that b is very small in comparison with the column height but it is proportional to the height. Therefore, it can be assumed that b = αδ where α = coefficient. This column tends to bend downstream in the presence of the bed shear stress.

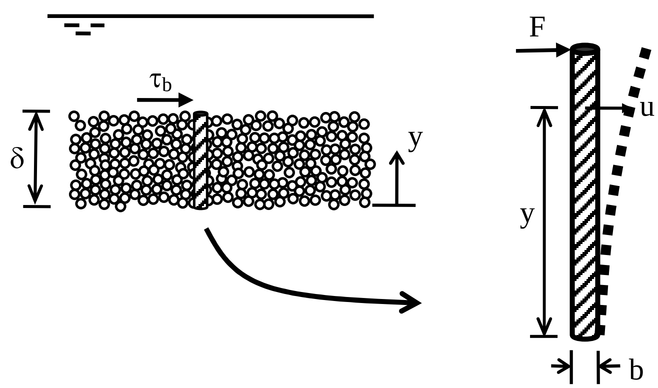

Fig. 1 Sketch of 'beam model' for sediment sheet flow.

At the location of y, which is measured from the bottom of the sheet flow, the horizontal displacement per unit time is denoted as u. From the theory of mechanics of materials (Case et al. 1999), the displacement distribution is expressed as

$$u = \frac{F}{6EI}\left(3\delta y^2 - y^3\right) \tag{1}$$

where F = concentrated force exerting at the upper end of the column; E = Young's modulus of elasticity; I = the second moment of the cross section about the principal axis; EI = flexural stiffness of the column. By integrating Eq. (1) from y = 0 to y = δ, we can express sediment transport per unit width as

$$q_v = \int_o^\delta u dy = \frac{F\delta^4}{8EI} \tag{2}$$

Since F ~ $\tau_b b^2 = \rho u_*^2 b^2$, where u_* = shear velocity, and b = αδ, Eq. (2) can be re-written as

$$q_v = \int_o^\delta u dy = C_1 \frac{\rho u_*^2 \delta^6}{EI} \tag{3}$$

where C_1 = coefficient. It can be seen that further applications of Eq. (3) depend on how the essential parameters, E, I and δ, are evaluated.

2.1 *Evaluation of Parameters*

Modulus

According to Hookean law, as sketched in Fig. 2(a), the modulus of elasticity E is equal to the ratio of the stress to the strain, $\sigma/(\Delta L/L)$, where σ = normal stress and $\Delta L/L$ = relative variations in the length of beam. Therefore, E has a dimension of stress. In comparison, one of fundamental properties for Newtonian fluid can be measured by the ratio of the shear stress to the shear rate, S_1, i.e., the dynamic viscosity (μ) of fluid for laminar flows (see Fig 2(b)). The viscosity can also be expressed as the product of the shear stress and the characteristic time, i.e.

$$S_1 = \mu = \rho \nu = \tau_b T_1 \tag{4}$$

Obviously, the characteristic time, T_1, for laminar flows near the bed can be represented by ν/u_*^2, where ν = kinematic viscosity and u_* = shear velocity.

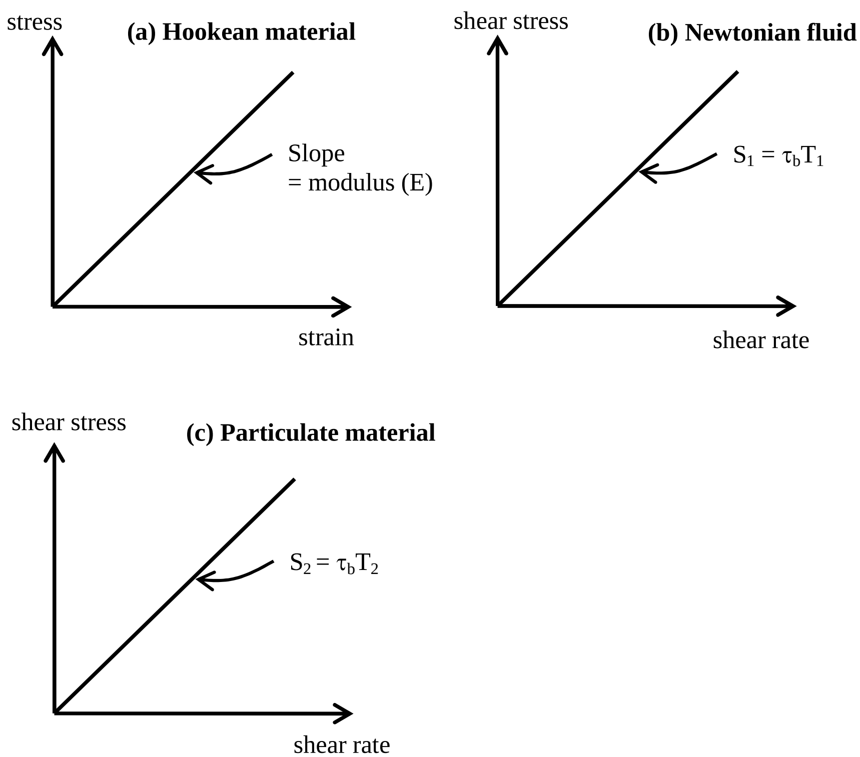

Fig. 2 Modulus of elasticity compared to shear-induced diffusion coefficients.

Similarly, for the case of elastic particulate flows, the ratio of the shear stress to the shear rate, S_2, can be assumed being proportional to the shear stress and the corresponding characteristic time, T_2 (see Fig. 2(c)). For sheet flows, the characteristic length for the collective particle motion is the thickness of the sheet layer, and therefore it may be further assumed that T_2 is proportional to δ/u_*. With this consideration, we get

$$S_2 = \tau_b T_2 = \rho u_* \delta \tag{5}$$

By comparing Fig. 2(a) and Fig. 2(c), the modulus 'E' applicable for the particulate material can be taken as the slope, S_2, i.e.

$$E = \rho u_* \delta \tag{6}$$

Second Moment

Since the dimension of the cross-section is b, and the second moment I is proportional to b^4, we may assume that

$$I = C_2 b^4 \tag{7}$$

where C_2 = coefficient.

Substituting Eqs. (6) and (7) into Eq. (3) gives

$$q_v = C_3 u_* \delta \tag{8}$$

Experimental studies by Wilson (1989) and Sumer et al. (1995) have suggested that the ratio of the thickness of the sheet flow to the particle diameter, δ/D, can be linearly related to the dimensionless shear stress, τ_*. With this result, Eq. (8) can also expressed as

$$q_b = \beta \frac{u_*^{\,3}}{\Delta g} \tag{9}$$

where $\Delta = (\rho_s - \rho)/\rho$; ρ_s = particle density; ρ = fluid density; g = gravitational acceleration and β = coefficient.

Eq. (9) indicates that the bedload transport rate for high shear stresses is independent of the size of sediment grains. This result may be against what we expect for other flow conditions, but it indicates that when sediment particles move in layers, the effect of sediment size can be ignored. In other words, for the sediment sheet flow, the average sediment velocity can be characterized by u_*, while the thickness of the mobile sediment layer can be characterized by $u_*^2/(\Delta g)$.

In Fig. 3, Eq. (9) is further compared with experimental data provided by Nnadi and Wilson (1992). In spite of two different particle sizes, Fig. 3 shows that all data points generally follow the same linear trend.

In the dimensionless form, Eq. (9) can be re-written as

$$\phi = \beta \tau_*^{\,1.5} \tag{10}$$

where $\phi = q_v/(\Delta g D^3)^{0.5}$ = dimensionless sediment transport rate. Actually, the power function given by Eq. (10) for the high-shear flow condition has been confirmed previously by several studies (see a summary provided by Cheng (2002)).

3. Discussion

Within the sheet flow, the sediment concentration decreases almost linearly from the top of sediment bed to the stationary layer (Wilson 1987). This variation definitely has some effects on the assumptions, which are made for the derivation of Eq. (1) and evaluations of the relevant parameters. However, such effects are not considered in the present study.

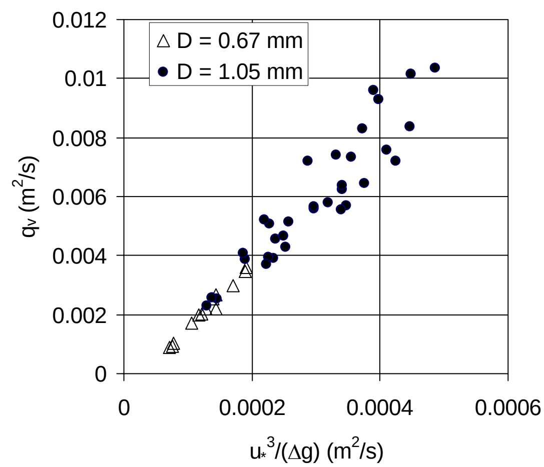

Fig. 3 Transport rates independent of particle diameters. Data were collected by Nnadi and Wilson (1992) for particles with specific gravity of 1.56.

On the other hand, the phenomena of sediment transport without bedforms also occur for laminar flows. Yalin and Karahan (1979) observed that sediment in laminar flows moves in layers at the initial motion condition. It is interesting to note that sediment transport over a flat bed in laminar flows has a weak dependence on the sediment size. Cheng (2004) demonstrated that for a flat bed, the dimensionless transport rate can be generally related to the dimensionless particle diameter, D_*, and the dimensionless shear stress, τ_*, as follows

$$\phi = 0.773 D_*^{2.66} \tau_*^4 \tag{11}$$

where $D_* = D(\Delta g/\nu^2)^{1/3}$. However, in the dimensional form, Eq. (11) can be changed to

$$q_v = 0.773 D^{0.16} \frac{u_*^8}{(\Delta g)^{2.61} \nu^{1.77}} \tag{12}$$

From Eq. (12), it can be seen that the power related to the particle diameter is much smaller than the others and thus the particle size effect is also insignificant.

4. Conclusions

A simple elastic model that is derived from the theory of beam deflection is used to evaluate sediment transport rates for the condition of high bed shear stresses. The collective sheet motion of sediment particles is shown to be independent of the particle size. Similar phenomena can be also found for sediment transport in laminar flows. This implies that for such sheet flow conditions, it may not be necessary to examine behaviors of individual particles.

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