SCOUR INDUCED BY HYDRAULIC JUMP: PRELIMINARY ANALYSIS*

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A theoretical and experimental investigation on the hydraulic jump when it is entirely positioned on a cohesionless bed is here presented. Various experimental conditions are reproduced, changing both the kinematic condition of the approaching flow and the size of bed material, in order to investigate the main geometrical features related to the scourhole. Experimental observations show that the movable character of the bed changes the nature of hydraulic jump drastically, in respect to the fixed bed case; in particular, it appears that the system eventually reaches an equilibrium configuration after a transitory period characterized by oscillating movements of hydraulic jump and bed profile between two quite different configurations. The experimental data are interpreted by employing momentum equation, in order to obtain predictive relationships for the maximum amplitude of scour at equilibrium.

1 Introduction

Hydraulic jump has been extensively studied in the case of fixed bed conditions in order to design river bed protection structures downstream of hydraulic devices such as gates, weirs and spillways. In such conditions, scour is mainly due to the residual turbulence developing downstream of the hydraulic jump. The problem has been studied by various authors both in the case of 'narrow' channels (Farhoudi and Smith, 1985; Hassan and Narayanan, 1984) and 'wide' channels (Dargahi, 2003).

Other authors have studied the scour mechanism occurring in hydrodynamic configurations similar to hydraulic jump (Wu and Rajaratnam, 1995): such as turbulent wall jets. According to Rajaratnam (1965), hydraulic jump may be interpreted as a plane turbulent wall jet under adverse pressure gradient and with a finite depth of flow .

Scour produced by submerged turbulent wall jets was studied by Rajaratnam (1981), Hassan and Narayanan (1986), Hogg et alii (1997) in the case of flow depth much greater than the thickness of the jet; Rajaratnam and Macdougall (1983) studied the same process in the case of shallow water with a depth approximately equal to the jet thickness. In the previous works, the process of scour has been interpreted in order to study the development of scour in time and to establish relationships among the maximum scour and average quantities related to the flow.

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In the present contribution an investigation devoted to study the scour profile when hydraulic jump occurs entirely on a movable bed, is presented.
The asymptotic flow and bed profile configurations have been studied by means of an experimental analysis and interpreted with momentum equation in order to obtain predictive equations for the maximum amplitude of the scour as a function of average kinematic characteristics of the approaching flow and the properties of bed material.
Experimental observations show that the movable character of the bed changes the nature of hydraulic jump drastically in respect to the fixed bed case; it appears that equilibrium configuration is reached after a transitory period characterized by quasi-periodic oscillation of the system.

2 Theoretical background

Momentum equation is applied to the asymptotic steady configuration of flow and bed profile; to this aim the geometry of bed profile is simplified by assuming the shape of the scour hole and the ridge of deposition to be triangular, as shown in Fig. 1.
The streamwise momentum balance per unit width applied between sections 1 and 2 (Fig. 1) and projected along the streamwise coordinate yields:

$$P_1 + P_u\big|_x - P_2 - P_d\big|_x + T_u\big|_x - T_d\big|_x = \rho q^2 (1/y_2 - 1/y_1)\,; \qquad (1)$$

where q is flow rate per unit width, ρ is flow density, y_1 and y_2 are flow depths at sections 1 and 2, P_1 and P_2 are total pressure forces at sections 1 and 2, P_u and T_u are forces due to total pressure and bed shear stress acting on the upstream face of the scour hole, P_d and T_d are forces due to total pressure and bed shear stress acting on the downstream face of the scour hole.

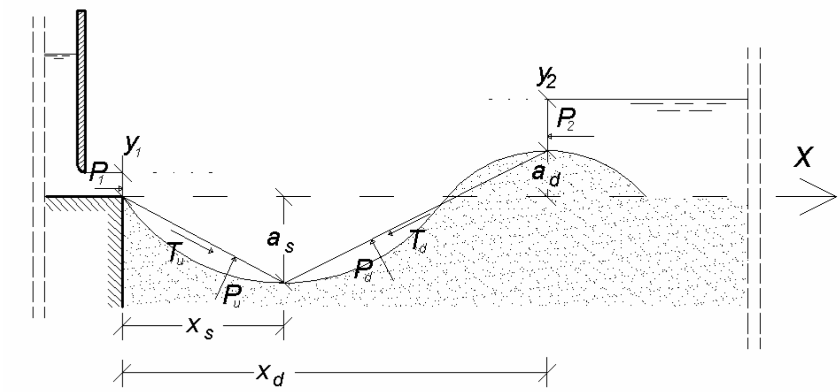

Figure 1. Definition sketch.

P_1 and P_2 are readily evaluated with hydrostatic law, while P_u is calculated with the following:

$$P_u = k\rho g \sqrt{\left(a_s^2 + x_s^2\right)}\left(y_1 + a_s/2\right); \qquad (2)$$

where a_s is the depth of maximum scour, x_s is the longitudinal coordinate of the point of maximum scour and k is a pressure correction coefficient equal to the ratio of the actual pressure on the upstream face of the scour hole to the hydrostatic pressure; this coefficient takes into account the effect of streamline curvature. The value assumed by k is known in the case of transition from supercritical to subcritical flow at an abrupt drop under a large variety of flow patterns (Ohtsu and Yasuda, 1991); in the present configuration k is unknown. P_d is total pressure force on the downstream face of the bed profile and it is evaluated by assuming an hydrostatic distribution of flow pressure, since in the downstream part of the scour hole streamlines appear to be relatively more aligned. T_u and T_d are assumed to represent the forces associated to the critical shear stress for the incipient motion of the sediment particles, here evaluated with the Shield criterion, on the upstream and downstream face of the bed profile, respectively.
The quantities related to main characteristics of asymptotic flow and bed profile configuration are evaluated by means of an experimental activity. The pressure correction coefficient k can then be calculated from Eq. (1)

3 Experiments

The experimental investigation has been carried out in the Hydraulic Laboratory of the Department of Civil Engineering at the University of Firenze, in a recirculating horizontal rectangular flume, 1.20 m long, 0.10 m wide with side walls in Plexiglas 0.70 m high (Fig. 2). Water enters the flume from a constant-head reservoir, under a vertical gate with smoothed edge and it leaves the flume passing over a Bazin type weir. The reach under investigation is 0.97 m long and it follows a 0.07 m long Plexiglas plate positioned immediately downstream of the vertical gate.
The experiments have been performed by feeding the flume with a constant liquid discharge equal to 2.3 l/s; no solid discharge have been provided. Various experimental conditions were reproduced, changing both the boundary kinematic condition of flow and the size of bed material, to investigate the main geometrical features related to the free surface and bed profile. The experiments carried out over movable bed have been repeated in the case of artificially roughened fixed bed. In particular, for each of the 5 well-sorted sediment mixtures employed, experiments have been carried out varying the height of the upstream vertical gate in order to obtain a supercritical flow with Froude number ranging from 2 to 9 at the upstream boundary while at the downstream boundary the height of tailwater has been adjusted to obtain, in the reach under investigation, both free and submerged jump. Values of median particle size D and of Froude number of the approaching supercritical flow Fr_1 are reported in Table 1.

Table 1. Main parameters of the run performed.

D [mm]	7.2	4.8	3.4	2.5	1.2
Fr_1	9	7	5	3	2

Measurements of water surface and bed profile were taken at equilibrium conditions; the macro-vortex structures of flow field were visualized by employing dye injection and video-camera.
This work presents results concerning only the case of free jumps (characteristic measurements of the equilibrium bed profile can be found in Doganieri et al., 2004).

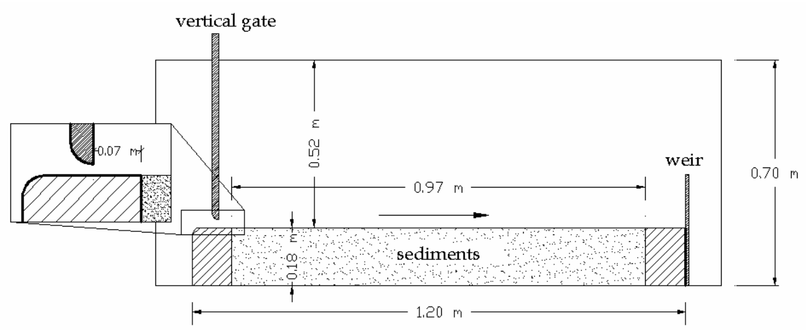

Figure 2. Experimental equipment (the arrow indicates the direction of main flow).

4 Results

4.1. *Configuration of hydraulic jump on a movable bed*

Experimental observations have shown that at equilibrium bed profile is characterized by a scourhole usually followed downstream by a ridge caused by the deposition of the eroded material. This shape of bed profile has been observed in all the experiments, except those with the finest size of sediments (D=1.2 mm); in this latter case bed particles were mainly transported in suspension and were able to leave the flume without showing any significant deposition phenomena in the reach under investigation; for this reason the data concerning D=1.2 mm are excluded from the present analysis.
Equilibrium configuration of flow field and bed topography is reached after a transitory period in which the system oscillates between two quite different configurations:

- configuration type I (Fig. 3a) characterized by an elongate form of the scour with relatively modest inclination of scour cavity; by the presence, towards the upstream boundary, of two macro-scale vortex structures, rotating in opposite directions, positioned near the free surface and the bed, leading to the formation of a standing stationary wave while, in the downstream part, the flow is relatively more aligned; by an average downstream sediment movement, apart from the upstream face of the scourhole where sediments are forced to move upstream;
- configuration type II (Fig. 3b) characterized by a scourhole with a relatively steep cavity; by the formation of a ridge inside the scourhole, leading to an increase of the free surface elevation in proximity of the vertical gate; by the formation of a counterclockwise rotating macro-vortex structure which pushes the main flow towards the bed; by an overall average movement of sediments towards downstream.

During the experiments, the system shifted periodically from one configuration to the other, usually with a decreasing frequency until when, after a time interval ranging from about 6 to 37 minutes, a stable configuration was reached, possibly due to the absence of sediment feeding. The equilibrium configuration reached by the system turned out to be configuration type I.

Such oscillating character of hydraulic jumps has also been pointed out by various authors (Ohtsu and Yasuda, 1991; Mossa et al., 2002; Dargahi, 2003) under different geometric conditions, such as downstream of spillways and at an abrupt drop.

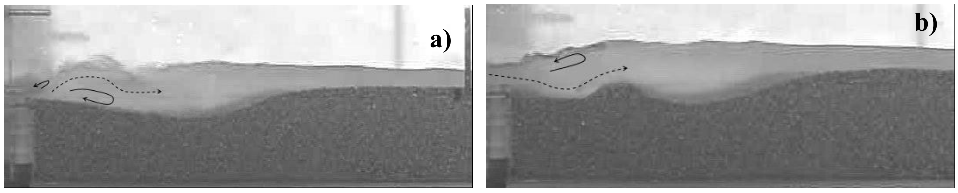

Figure 3. Side view of the free surface and bed profile configuration type I (a) and II (b). Continuous arrows indicate the pattern of macro-vortex system, dashed lines the direction of main flow.

4.2. *Scour depth equations*

Measurements of equilibrium bed profile are employed in order to solve the momentum equation (1) for the pressure correction coefficient k.

Results are plotted as a function of densimetric Froude number of the flow at the upstream boundary $Fd_1=(q/y_1)/(\Delta gD)^{1/2}$ with Δ=relative submerged sediment density. This parameter has been introduced by Rajaratnam (1981) while analyzing the erosion caused by a submerged plane wall turbulent jet.

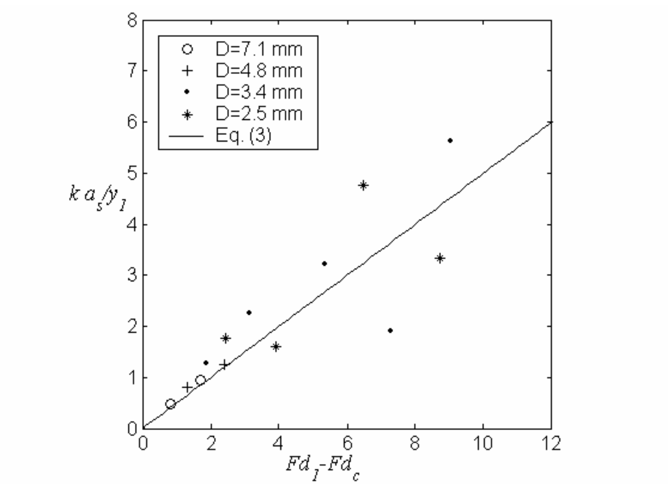

Figure 4. The product between the pressure correction coefficient k and the dimensionless maximum scour as/y_1 as a function of Fd_1-Fd_c.

In Fig. 4 the dimensionless parameter $k\, a_s/y_1$ is shown as a function of the parameter Fd_1-Fd_c; Fd_c is the critical value of the densimetric Froude number for the incipient motion of bed material, here estimated with Shields criterion. The quantity $k\ a_s/y_1$ appears to increase almost linearly with Fd_1-Fd_c ; when Fd_c approaches its critical value the scour vanishes and $k\, a_s/y_1$ tends to zero. The data points do not seem to be explicitly influenced by the size of bed material and are interpolated with the following linear curve:

$$k\frac{a_s}{y_1} = 0.5\left(Fd_1 - Fd_c\right). \tag{3}$$

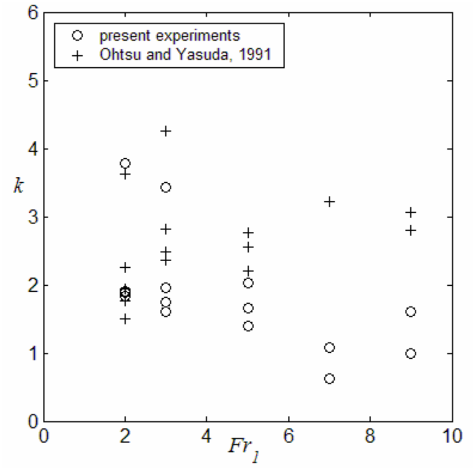

Figure 5. The pressure correction coefficient k as a function of the Froude number of the flow at the upstream boundary Fr_1.

In fig. 5 the pressure correction coefficient k is reported as a function of Froude number Fr_1 of the upstream supercritical flow. A comparison with k values estimated by Ohtsu and Yasasuda (1991) in the case of the formation of hydraulic jump at an abrupt drop is also shown; in their configuration k, evaluated on step face of the drop, can assume quite different values depending on flow patterns and on inflow and downstream boundary conditions. The present case takes into account results concerning flow pattern known as wave jump, characterized by the formation of a large standing eddy just below the drop and by the flow being a normal open-channel flow at a short distance downstream of the eddy, and low drop conditions ($0.5\div1.5 \le a_s/y_1 \le 8.0\div9.0$; $1.0 \le Fr_1 \le 5.0$). It appears that the pressure on the upstream face of scourhole is larger than the hydrostatic pressure in almost all the runs; such difference is greater when $Fr_1 \le 5$; in this range, the calculated values of k are similar to those estimated by Ohtsu and Yasasuda (1991) though in quite different conditions. When $Fr_1 > 5$ the present runs exhibit smaller values of k, not larger than 2, and they are quite different from those predicted by Ohtsu and Yasasuda (1991). In order to predict the maximum amplitude of scour a_s at equilibrium caused by the hydraulic jump with Eq. (1), further relationships between known characteristic quantities of the upstream supercritical flow and downstream quantities such as the flow

depth y_2 and the height of the deposit a_d need to be introduced. To this purpose the theoretical subcritical flow depth y_{2t}, conjugate of y_1, in the case of free hydraulic jump over a fixed bed is calculated by means of Belanger equation.
In Fig. 6 the ratio between y_{2t} and measured y_2 is reported as a function of Fd_1-Fd_c. It appears that y_{2t}/y_2 is an increasing function of the parameter Fd_1-Fd_c, being about 1 when Fd_1 approaches Fd_c; in this condition no bed deformation takes place and the hydraulic jump is the same as in the fixed bed case.

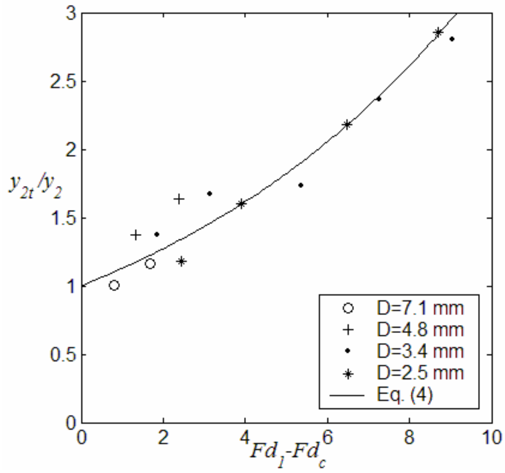

Figure 6. The ratio between theoretical flow depth y_{2t} in the case of fixed bed condition and the measured flow depth y_2 as a function of Fd_1-Fd_c.

The data points in Fig. 6 are interpolated with the following equation:

$$\frac{y_{2t}}{y_2} = e^{0.12(Fd_1 - Fd_c)} \tag{4}$$

Finally the height of the deposit a_d, made dimensionless with the flow depth y_1, is expressed through a linear function (Eq. 5) of the ratio y_{2t}/y_2 as shown in Fig 7. Note that the height of deposit a_d vanishes when y_{2t}=y_2.

$$\frac{a_d}{y_1} = 1.6\left(\frac{y_{2t}}{y_2} - 1\right) \tag{5}$$

By neglecting the total shear stresses acting on the faces of the scourhole, which play a second order role, momentum equation (1), with the help of Eqs. (3,4,5), can then be employed in order to predict the maximum scour a_s for given kinematic conditions of the flow at upstream boundary and size of bed material.

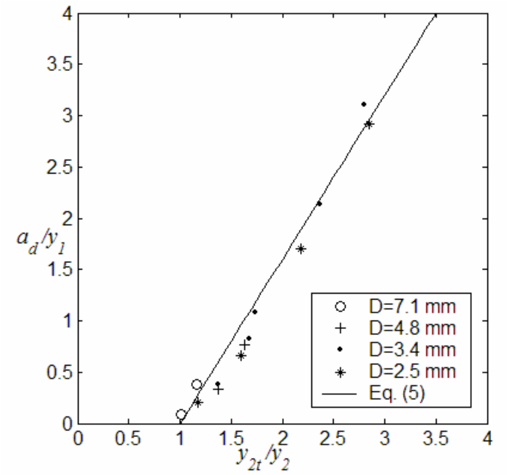

Figure 7. The ratio between measured deposit height a_d and upstream flow depth y_1 as a function of y_{2t} / y_2.

5 Conclusions

An experimental analysis of hydraulic jump on a movable bed in a flume fed with a constant liquid discharge only is presented. The runs have shown that an equilibrium configuration is attained after a transitory period in which the system experiences quasi-periodic oscillation between two quite different configurations.
Measurements of equilibrium free surface and bed profile have been theoretically interpreted in order to obtain predictive relationships for the amplitude of maximum scour at equilibrium as a function of the kinematic characteristics of the incoming supercritical flow and bed material size. In particular, for the momentum equation to be satisfied, the pressure on the upstream face of the scourhole must be greater than the hydrostatic pressure, as suggested by Ohtsu and Yasuda (1991) in the case of the formation a wave jump at an abrupt drop. Finally it is shown that some characteristic geometrical features of bed profile can be expressed as a function of densimetric Froude number of the flow at the upstream boundary; when the latter approaches its critical value for the onset of motion of bed material, the erosion and deposition processes vanish and fixed bed conditions are attained.

Acknowledgments

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