SCOUR AT THE CHANNEL CONTRACTIONS IN THE GRAVEL-BEDS

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Scour in the channel contraction occurs due to flow concentration within the channel contractions inducing excessive bed shear stress. This paper presents a mathematical model to estimate the maximum scour depth within the channel contractions. The model is based on the energy and continuity equations between the sections before and within the channel contraction. The model is capable of estimating the maximum scour depths for clear-water and live-bed conditions. However, it is observed that no attention is paid so far to estimate the scour depth in the gravel-beds. Hence, in this study, an attempt is made to investigate the magnitude of maximum scour depth within the channel contractions in the gravel-beds. The laboratory experiments were conducted using the gravels in the range of 4 – 14.25 mm and sands in the range of 0.81 – 2.54 mm for the models with percentage of contractions between 30 and 60. It is found that the results of the mathematical model are in good agreement with the experimental results.

1 Introduction

The reduction in the width of river or channel, to minimize the cost of the structures that are built across, is called river contraction or channel contraction. Bridges, barrages, weirs, and cross-drainage works are the common structures constructed across the rivers at which the river width is reduced. Also, cofferdams and end dump channel constriction used for the maintenance of the riverbanks are the other examples of channel contraction. Channel contractions are designated as long or short based on the ratio of the length of contraction $L$ to the approaching channel width $b_1$. As per Komura (1966), a channel contraction is considered long when $L/b_1 > 1$, whereas according to Webby (1984) the ratio $L/b_1 > 2$. However, in the present experimental investigation, it is confirmed that $L/b_1 \geq 1$ is adequate for a contraction to be considered long.

The reduction in the flow area of the channel increases the velocity of flow in the contracted zone of the channel. As a result, the bed shear stress induced by the flow increases considerably causing the scour of sediment bed within the channel contraction. Such localized scour in the contracted zone of the channel is called contraction scour. Straub (1934) initiated the study of scour in long contractions and proposed a simplified one-dimensional theory. His investigation was later extended and modified by Ashida (1963), Laursen (1963), Komura (1966), Gill (1981) and Webby (1984). Lim (1993) put forward an empirical equation of maximum equilibrium scour depth in long contractions under clear-water and live-bed scour conditions. In most of the aforementioned studies, the investigations were undertaken only on sediment beds. However, no attention is paid so far to estimate the scour depth in the gravel-beds in long contractions.

The present study aims at the development of a mathematical model for the computation of clear-water scour depth in long contractions, based on the continuity and energy equations, with and without sidewall correction for contracted zone. On the other
hand, the live-bed scour depth is derived, introducing the sediment continuity equation. The models are validated by the present and others experimental data.

2 Experimentation

Experiments were carried out in a tilting flume of 0.6 m wide, 0.7 m deep and 12 m long. Four contraction models made of perspex sheets with percentage of contractions between 30 and 60 were used in this study. Contraction models were having uniform contraction zone of 1 m length (Fig. 1) with smooth upstream and downstream transitions. The models were symmetrically attached to the side glass-walls of the flume with the upstream end located at 6 m from the flume inlet in order to have a fully developed turbulent flow at the entrance of the contraction. The size of sediment recess was 3 m long, 0.6 m wide and 0.3 m deep. Eight uniform sediment (three sand and five gravel sizes) having geometric standard deviation $\sigma_g < 1.4$ (Dey et al. 1995) were used in the experiments. The flow discharge, controlled by an inlet valve, was measured using a calibrated rectangular weir fitted at the outflow channel, where the water from the flume was discharged. The flow depth in the flume was adjusted by a downstream tailgate. The flume was initially filled with the water until the desirable flow depth was reached to avoid the undesirable scour due to sheet flow. Then the discharge in the flume was gradually increased to the desired value corresponding to the clear-water scour condition, and the experiment was run till the equilibrium scour reached. The experiments were run with the average approaching flow velocities $0.98U_c > U_1 > 0.9U_c$ being achieved by adjusting the discharge and tailgate, where $U_1 =$ average approaching flow velocity and $U_c =$ critical velocity for sediments, calculated using the following semi-logarithmic average velocity equation (Lauchlan and Melville, 2001).

![Figure 1. Schematic view of a channel contraction at equilibrium scour condition: (a) top view and (b) side view.](image)
\[ \frac{U_c}{u_c} = 5.75 \log \left( \frac{h_1}{2d_{50}} \right) + 6 \]  

(1)

where \( h_1 \) = approaching flow depth; \( d_{50} \) = median sediment diameter and \( u_c \) = critical shear velocity for sediments, obtained from Shields diagram. Gill (1981) indicated that the equilibrium scour depth in long channel contractions becomes maximum when \( U_1/U_c \rightarrow 1 \) under clear-water scour. But, it is important to mention that the approaching channel bed becomes disturbed by the approaching flow when \( U_1 \rightarrow U_c \). Therefore, the experiments were conducted under a clear-water scour condition, maintaining \( U_1/U_c < 1 \) or \( u_1/u_c < 1 \); where \( u_1 \) = approaching shear velocity. The experiments were run for a period of more than 24 h until an equilibrium state of scour was achieved, which was ascertained when negligible difference of scour depth was observed at an interval of two hours after 24 h. In this study, the equilibrium scour depths \( d_s \) were observed near the centerline within the contracted zone. The range of data of present study and other investigators, used for the model validation, are given in Table 1.

<table>
<thead>
<tr>
<th>Scour type</th>
<th>Source</th>
<th>( d_{50} ) (mm)</th>
<th>( \bar{b} )</th>
<th>( U_1/U_c )</th>
<th>( h_1 ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear-water</td>
<td>Present study</td>
<td>0.81 - 14.25</td>
<td>0.3 - 0.7</td>
<td>0.90 - 0.98</td>
<td>8 - 14</td>
</tr>
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<td></td>
<td>Komura (1966)</td>
<td>0.35 - 0.55</td>
<td>0.25 - 0.5</td>
<td>0.95 - 1.00</td>
<td>2.8 - 8.4</td>
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<tr>
<td></td>
<td>Gill (1981)</td>
<td>0.92 - 1.53</td>
<td>0.66</td>
<td>0.91 - 0.97</td>
<td>4.4 - 6.9</td>
</tr>
<tr>
<td></td>
<td>Lim (1993)</td>
<td>0.47</td>
<td>0.3 - 0.65</td>
<td>0.96 - 1.00</td>
<td>2.4 - 2.8</td>
</tr>
<tr>
<td>Live-bed</td>
<td>Komura (1966)</td>
<td>0.55</td>
<td>0.25 - 0.5</td>
<td>1.04 - 1.22</td>
<td>4.7 - 7.5</td>
</tr>
<tr>
<td></td>
<td>Gill (1981)</td>
<td>0.92 - 1.53</td>
<td>0.66</td>
<td>1.02 - 1.52</td>
<td>3.4 - 7.9</td>
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<tr>
<td></td>
<td>Lim (1993)</td>
<td>0.47</td>
<td>0.3 - 0.65</td>
<td>1.07 - 1.27</td>
<td>3.1 - 4.9</td>
</tr>
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3 Mathematical Model

3.1 Energy and Continuity Equations

Applying the energy and continuity equations between sections 1 and 2 for the flow situation at equilibrium scour condition (Fig. 1) yields

\[ h_1 + \frac{U_1^2}{2g} = h_2 + \frac{U_2^2}{2g} - d_s + h_f \]  

(2)

and

\[ U_1 h_1 b_1 = U_2 h_2 b_2 \]  

(3)
where $h_2 =$ flow depth in contracted zone; $U_2 =$ flow velocity in contracted zone; $h_f =$ head loss between sections 1 and 2; and $b_2 =$ contracted width of channel. If the reach of contraction is a long and gradual one, the resulting head loss $h_f$ is negligible (Graf 2003).

### 3.2 Clear-water Scour Model

**Determination of Scour Depth with Sidewall Correction**

In case of clear-water scour, the equilibrium scour depth $d_s$ reaches in a long contraction, when the flow velocity $U_2$ in the contracted zone attains critical velocity $U_c$ for sediments. The flow velocity $U_2|_{U_2=U_c}$ in the contracted zone can be obtained from the following equation of bed shear stress as a function of dynamic pressure.

$$U_2|_{U_2=U_c} = \frac{2.828}{f_b^{0.5}} \cdot U_c$$

where $f_b =$ friction factor associated with the bed. The Colebrook-White equation, used to evaluate $f_b$, is

$$\frac{1}{\sqrt{f_b}} = -0.86 \ln \left( \frac{k_s P_b}{14.8 A_b} + \frac{2.51}{R_b \sqrt{f_b}} \right)$$

where $k_s =$ equivalent roughness height [= $2d_{50}$]; $A_b =$ flow area associated with the bed; $P_b =$ wetted perimeter associated with the bed [= $b_2$]; and $R_b =$ flow Reynolds number associated with the bed [= $4U_2|_{U_2=U_c} A_b/(\nu P_b)$].

In the contracted zone, the bed is rough consisting of sediment particles and the sidewalls are smooth. Therefore, the friction factor associated with the wall $f_w$ is considerably different from $f_b$. Hence, Vanoni’s (1975) method of sidewall correction is applied here for the contracted zone of the channel, owing to the smooth wall and rough bed, as was done by Dey (2003a,b). The solution for $f_b$ was obtained from the following equations:

$$f_b = 0.316 R_p \left( \frac{4U_2|_{U_2=U_c} A}{\nu P_w} - \frac{R_b P_b}{P_w} \right)^{-1.25}$$

$$\frac{1}{\sqrt{f_b}} = -0.86 \ln \left( \frac{k_s U_2|_{U_2=U_c}}{3.7 \nu R_b} + \frac{2.51}{R_b \sqrt{f_b}} \right)$$

where $A =$ total flow area [= $h_2 b_2$]; and $P_w =$ wetted perimeter associated with the wall [= $2h_2$]. In clear-water scour, the continuity equation becomes
For a given data of $U_1$, $h_1$, $b_1$, $b_2$ and $d_{50}$, the unknowns $U_2\big|_{U_2=U_\epsilon}$, $h_2$, $R_b$, and $f_b$ can be determined numerically solving Eqs. (4) and (6)-(8). Then, Eq. (2) is used to determine equilibrium scour depth $d_s$ as given below:

$$d_s = h_2 + \frac{U_2^2\big|_{U_2=U_\epsilon}^2}{2g} - h_1 - \frac{U_1^2}{2g}$$  \hspace{1cm} (9)

The comparison of nondimensional equilibrium scour depths $\hat{d}_s \left[= d_s/h_1\right]$ computed from Eq. (9) with the experimental data is shown in Fig. 2(a) having a correlation coefficient 0.95, which indicates that the model corresponds closely with the experimental data.

### Determination of Scour Depth without Sidewall Correction

This is a simplified approach, where the average flow velocity in the contracted zone $U_2\big|_{U_2=U_\epsilon}$ for equilibrium scour is determined using the equation of semi-logarithmic average velocity as

$$\frac{U_2\big|_{U_2=U_\epsilon}}{u_e} = 5.75 \log \frac{h_2}{2d_{50}} + 6$$  \hspace{1cm} (10)

For a known data set of $U_1$, $h_1$, $b_1$, $b_2$ and $d_{50}$, the unknowns $U_2\big|_{U_2=U_\epsilon}$ and $h_2$ can be obtained numerically solving Eqs. (8) and (10) and then, equilibrium scour depth $d_s$ is determined from Eq. (9). The comparison between the nondimensional equilibrium scour depths $\hat{d}_s$ computed from this method and the experimental data is shown in Fig. 2(b). The correlation coefficient 0.96 for the computed and the experimental data indicates that the model fits excellently with the experimental data.

### 3.3 Live-bed Scour Model

In live-bed scour, the equilibrium scour depth $d_s$ reaches, when the sediment supplied by the approaching flow into the contracted zone is balanced by the sediment transported out of the contracted zone. Thus, at equilibrium, the sediment continuity equation between sections 1 and 2 is

$$\xi \big|_{u_c=u_{c_1}} b_1 = \xi \big|_{u_c=u_{c_2}} b_2$$  \hspace{1cm} (11)

where $\xi$ = bed-load transport of sediments. The bed-load transport $\xi$ of sediments can be estimated by the formula of Engelund and Fredsøe (1976) as
Figure 2. Comparison between the experimental data and computed values of equilibrium scour depths $d_s^*$ using clear-water scour model (a) with sidewall correction and (b) without sidewall correction.
\[ \xi = 1.55 \pi \frac{d_{50}}{u_0} u^* \left( 1 - 0.7 \frac{\Delta u_{c}}{u^*} \right) \left[ 1 + \left( \frac{0.085 \pi \Delta g d_{50}}{u^* - u_{c}} \right)^2 \right]^{0.25} \] (12)

Assuming the semi-logarithmic average velocity for approaching flow, the shear velocity \( u^* \) at section 1 is obtained as

\[ u^* = U_1 \left( 5.75 \log \frac{h_1}{2d_{50}} + 6 \right)^{-1} \] (13)

In the contracted zone, incorporating the semi-logarithmic average velocity in Eq. (3), one obtains

\[ \frac{h_1}{h_2} \cdot \frac{h_1}{h_2} = \frac{U_2}{U_1} \left( 5.75 \log \frac{h_2}{2d_{50}} + 6 \right) \] (14)

For a given data of \( U_1, h_1, b_1, b_2 \) and \( d_{50} \), the unknowns \( U_2 \) and \( h_2 \) can be determined numerically solving Eqs. (11), (13) and (14). The Eq. (2) is used to determine equilibrium scour depth \( d_s \) as given below:

\[ d_s = h_2 + \frac{U_2^2}{2g} - h_1 - \frac{U_1^2}{2g} \] (15)

The comparison of nondimensional equilibrium scour depths \( \hat{d}_s \) computed using the model with the live-bed scour data is shown in Fig. 3 having a correlation coefficient 0.83, which indicates that the live-bed scour model agrees reasonably with the experimental data.

4 Conclusions

The paper presents a mathematical model for the computation of clear-water scour depth in long contractions based on the continuity and energy equations, with and without sidewall correction for contracted zone. Also, the model is extended to the live-bed scour introducing the sediment continuity equation. The scour depths computed using the models are in excellent agreement with the experimental data.
Figure 3. Comparison between the equilibrium scour depths $d_{50}$ computed using live-bed scour model and the experimental data.

References


