PREDICTION OF TOTAL SEDIMENT DISCHARGE

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Following Bagnold's approach, a relationship between sediment transport and energy dissipation is developed. The major assumption made in the study is that the near bed velocity plays a dominant role in the process of sediment transport. A general relationship between the energy dissipation and sediment transport is first proposed. Then the equations for total sediment transport are derived by introducing the appropriate expression of energy dissipation rate. This formula has the advantages of accuracy, ease of computation and applicable to a wide range of physical problems. The total sediment discharge, g_t is linearly related to a total-load transport parameter, $T_T = \tau_o \left(u_*'^2 - u_{*c}^2\right)/\omega$. The latter involves variables that can be easily measured in the field or laboratory, i.e., flow depth, mean flow velocity, energy slope, median sediment size and density, and water temperature. The writers used more than 3500 published total-load data from field and flume studies, and the results showed that 84% of the data were predicted within the a magnitude of 2 times the measured values. This is an encouraging achievement considering the large database and the range of variables covered by the formula. Within the flows investigated, the derived relationships are fairly and evenly consistent with the available data over a wide range of conditions.

1 Introduction

The mechanism of sediment transport has been an important subject in the design and operation of canal systems, river training and regulation. To date, there are many formulas for estimating the rate of sediment discharge. There are basically three classes of formulas: i.e., bed-load, suspended load and total load formulas. Under natural flowing conditions, no sharp distinction between the bed-load and suspended mode of transport is discernible, this is particularly true in situations of heavily sediment laden flow, whence the suspended load and the bed-load cannot be separated (Chien and Wan, 1999). Thus, total load formulas are useful to compute the sediment discharge in natural rivers.

Chien and Wan (1999) provided a good summary of the well-cited equations, such as those proposed by Einstein (1942), Meyer-Peter and Muller (1948), Bagnold (1966), Yalin (1977), Engelund and Hanson (1972), and Ackers and White (1973). Chien and Wan had shown that all these equations can be expressed as Ö = f(ø), i.e., the dimensionless sediment transport rate.

$$\Phi = \frac{g_t}{\gamma_s} \left(\frac{\gamma}{\gamma_s - \gamma} \right)^{\frac{1}{2}} \left(\frac{1}{gd_{50}^3} \right)^{\frac{1}{2}} \tag{1}$$

is a function of the Shields shear stress parameter,

$$\Psi = \left(\frac{\gamma_s - \gamma}{\gamma} \right) \frac{d_{50}}{RS} \tag{2}$$

where Ô = Einstein's sediment intensity parameter; d_{50} = medial sediment size; g_t = sediment transport rate per unit width in kg/m/s; γ_s and γ = the specific weight of sediment and water, respectively; R = hydraulic radius; S = energy slope; g = gravitational acceleration. There are other parameters developed to express sediment discharge or concentration, such as the $V^3/(gR\omega)$ proposed by Velikanov (1954), the dimensionless unit stream power, VS/ω by Yang (1996), and the transport-stage parameter, $T = (u_*^{'2} - u^2{}_{*c}) / u_{*c}^2$ by Van Rijn (1982), in which V = depth-averaged velocity, ω = sand fall velocity and u'_* = $(g^{0.5}/C')V$ = bed-shear velocity related to grains; C' = $18\log(12R/3d_{50})$ = Chezy-coefficient related to grains; u_{*c} = critical shear velocity.

The objectives of this paper are: (1) to investigate the relationship between these empirical parameters and the measured sediment transport rate; (2) to develop a general equation of sediment transport; and (3) to investigate the accuracy of the new equation with experimental data.

THEORETICAL CONSIDERATIONS

Bagnold (1966) developed a sediment transport function from the power concept. The total load, g_t is expressed in the following way from the power concept

$$g_t = \frac{\gamma_s}{\gamma_s - \gamma} k_1 E \frac{u_s}{\omega} \tag{3}$$

where u_s = mean transport velocity of sediment, $k_1 = \omega e_b / (u_s \tan\alpha) + 0.01$, in which e_b = efficiency coefficient; taná = ratio of tangential to normal shear force.

Bagnold (1966) expressed the available energy as follows

$E = \hat{o}_o V$

(4)

Yalin's (1977) comments are that Eq. 4 means the loss of potential energy of the flow per unit area and time. Different from Bagnold's assumption, some researchers found empirically that the sediment transport is closely related to the energy dissipation near the bed (Yalin, 1977), i.e. the product of bed shear stress $\hat{o}_o$ and near bed velocity u_* with the following form

$$E = \tau_o u_* \tag{5}$$

where u_* = $(gRS)^{0.5}$ = $(\hat{o}_o/\tilde{n})^{0.5}$. In Eq. 5, the near bed velocity is represented by the over-all shear velocity, u_*. According to Einstein that the total shear stress, $\hat{o}_o$ can be divided into two components, viz. bed-shear related to grains, $\tilde{n}u_*^{'2}$ and sand wave shear. Van Rijn (1984) found that a better relationship can be obtained between sediment transport and the bed-shear velocity related to grains, u'_* because it can eliminate bed form roughness, and also it is simple and convenient. Van

Rijn (1984) comprehensivly investigated the sediment transport phenomenas and concluded that the bed-shear velocity related to grains rather than the total shear velocity, u_* or mean velocity, V plays a dominant role for sediment transport.

Therefore, it is acceptable to express the "available energy", E in Eq. 3 in the following way.

$$E = \tau_o u_*' \tag{6}$$

The mean transport velocity of sediment, u_s in Eq. 3 is defined as follows (Yalin 1977).

$$u_s = \frac{1}{\lim_{\varepsilon \to 0} \int_\varepsilon^h c_y dy} \lim_{\varepsilon \to 0} \int_\varepsilon^h c_y u dy \tag{7}$$

where y = distance from the bed, u = local velocity at level y, c_y = volumetric concentration of particles at level y. Bagnold (1966) assumed that u_s = V = $\int_0^h u dy/h$, this is equivalent to the assumption that the concentration c_y does not vary with y as Yalin (1977) commented.

As is well known, the sediment concentration increases from zero at water surface with the decreasing values of y, in other words, most sediment particles travel with water near the bed, therefore, it is rational to assume that the mean transport velocity of sediment is proportional to the near bed water velocity, i.e.

$$u_s = \alpha_1 u_*' \tag{8}$$

in which α_1 = coefficient.

Substituting Eqs. 8 and 6 into Eq. 3, one obtains Yang and Lim's (2003) formula for sediment transport.

$$g_t = k \frac{\gamma_s}{\gamma_s - \gamma} \tau_o \frac{u_*'^2 - u_{*c}^2}{\omega} = k \frac{\gamma_s}{\gamma_s - \gamma} T_T \tag{9}$$

where $u_*' = 2.5 \ln \frac{11R}{2d_{50}}$ (10)

The procedure of calculation is as follows:

1. Determine u_{*c}^2 and ω based on d_{50}, ρ and ρ_s.
2. Calculate the grain shear velocity $u_*'^2$ using Eq. 10.
3. Calculate the mean bed shear stress, $\tau_o = \gamma RS$.
4. Use k = 12.5 in (9).

Data for Verification

Almost 2500 data sets used for the verification of Eq. 9 were compiled by Brownlie (1981). The large number of data sets were sub-divided and plotted in Figs 1 - 3 in the format of predicted sediment discharge against the measured values where the solid lines represent prefect agreement and the dotted lines represent ±100 % discrepancy. The main objective is to study the influence of the various parameters in Eq. 9 on the constancy of k and also to achieve clearer presentation.

Fig. 1 shows the comparison based on a total of 341 data sets from 10 different researchers. The hydraulic conditions of these data are summarized in Table 1. The good agreement as shown in Fig. 1 testifies that k = 12.5 is indeed a constant.

Figure 2 includes only Gilbert's (1914) data. As water temperature was not included in the record, the writers assumed that the temperature was 20°C, and used that value in the calculations. A total of 763 out of 774 data points were used in the plot after excluding data with ($u_*'^2$ - u_{*c}^2) < 0, and those with the measured concentration less than 10 ppm. It is to be expected that the assumption of constant temperature would result in some errors, especially for fine sand as a result of the influence of viscosity on the fall velocity. Indeed, the results showed that the agreement is better for coarse sand than the fine sand.

Fig. 3 shows a plot with 1256 data sets from various researchers. The uniqueness of this plot is the good agreement obtained using Eq.9 for experiments conducted with high sediment concentrations, see Table 2. It is to be highlighted that the data set includes partial experiments conducted with hyper-concentrated flow. Figure 3 shows that the predictions are well within the ±100 % discrepancy lines, which is very encouraging.

Influence of Sediment Size on k

The sediment sizes for the data presented in Figs. 1-3 are in the range of 0.088 mm to 6 mm. To investigate the influence of sediment size on k, Fig. 4 is specifically plotted using data with gravel having d_{50} of up to 28.65 mm, the predictions were good with the same k value in Eq. 9.

On the other extreme, Eq. 9 is also tested to see if it can be applied to predict sediment discharge with very fine bed material that is generally regarded as wash load (sediment size is finer than 0.07mm, Partheniades, 1977). Fig. 5 shows the predictions for laboratory data with d_{50} = 0.011 mm from Kalinske and Hsia (1945) and field data with d_{50} = 0.02 to 0.07 mm from Indian Canals by Chitale (1966). As a comparison, other equations are also included in Fig. 5 and it can be seen that Eq. 9 provides a reasonable prediction. Hence, it can be concluded that wash-load could also be predicted since the motions for coarse and fine sediment are governed by identical physical laws, Partheniades (1977).

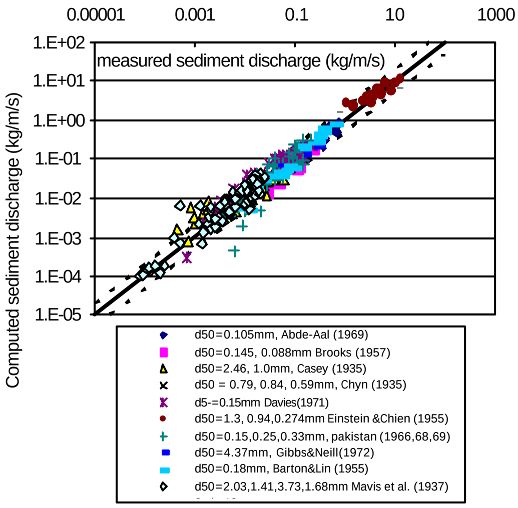

Fig. 1 Comparison of total sediment discharge computed using (9) and measured data from different researchers

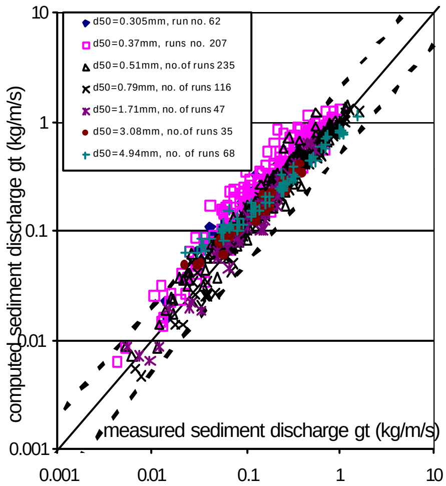

Fig. 2 Comparison of total sediment discharge computed using Eq, 9 and Gilbert's (1914) data

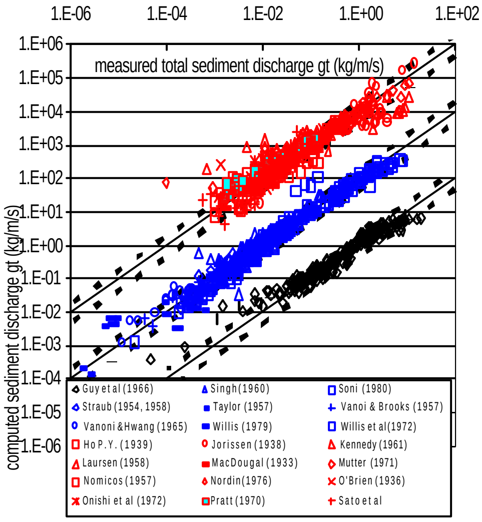

Fig. 3 Comparison of total sediment discharge computed using Eq. 9 and various researchers' measured data.

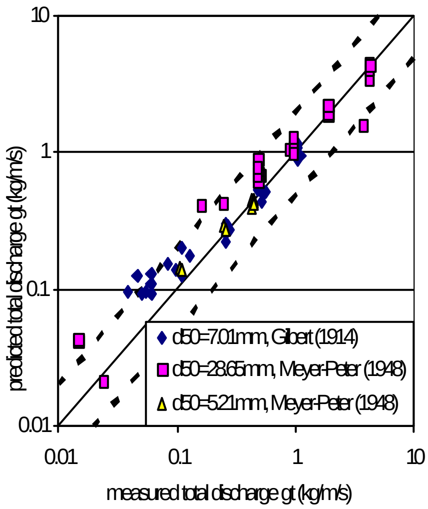

Fig. 4 Comparison of total sediment discharge computed using Eq. 9 and measured transport data for gravels

Influence of Channel Aspect Ratio on k

Williams' (1970) work on the effects of channel aspect ratio on sediment transport is by far one of the most comprehensive. He conducted 177 experiments using four different flumes with widths, b = 7.6, 15.2, 30.4 and 60.9 cm. For each flume, tests were conducted for four flow depths, i.e., h = 3.0, 9.1, 15.2 and 21.3 cm. The minimum aspect ratio, b/h tested is 0.33.

For the present purpose of testing the influence of channel aspect ratio on k, calculation for u_*' in Eq. 9 was done for each run with and without sidewall corrections. The results are shown in the first two rows in Table 3. As expected, the predictions with sidewall correction are better. However, the accuracy (see % scores) for the results without sidewall correction is also acceptable, see Fig. 6.

For practical purpose, the writers are of the opinion that Eq. 9 is of sufficient accuracy even without sidewall correction. This is an interesting observation, as most formulas would require sidewall correction to account for the 3-D effect of the flow. Williams' data are excellent in that it shows how other formulas fare for channel with small aspect ratio. The writers have chosen the formulas proposed by Bagnold (1966), Dou (1974), Yang (1996) and van Rijn (1984) for this purpose, and the results are tabulated in Table 3, and expressed in terms of a discrepancy ratio defined as:

$$r = \frac{g_{t(measured)}}{g_{t(computed)}} \qquad (12)$$

For all data, Eq. 9 depicts the best score (with and without sidewall correction) when compared to these other formulas. This is a significant conclusion as it illustrates that Eq. 9 is able to predict with sufficient accuracy even for b/h = 0.33 without sidewall correction. This finding is crucial and is a major improvement to the prediction of total sediment discharge.

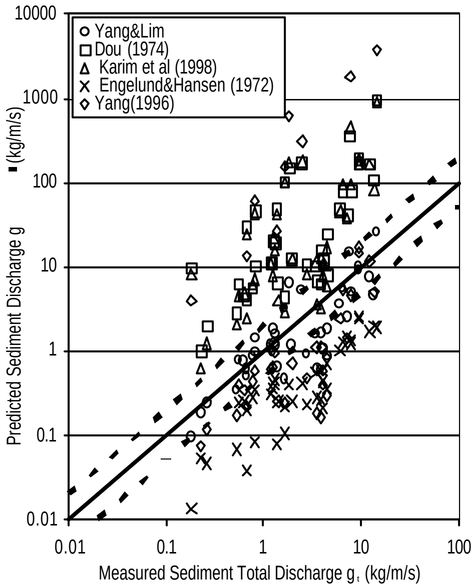

Fig. 5 Comparisons between measured silt discharge and prediction using Eq. 9 and other formulas

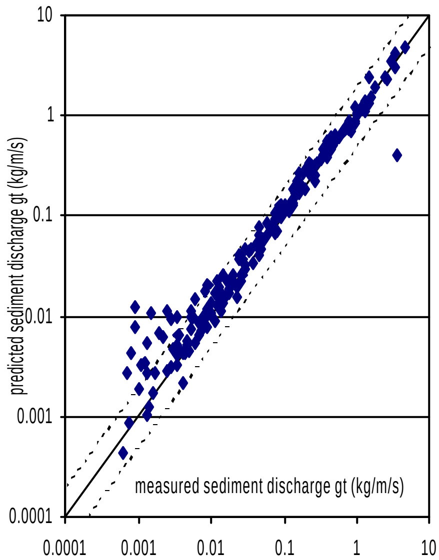

Fig. 6, Effect of channel aspect ratio on the predictability of Eq. 9, based on Williams' (1970) data.

Conclusions

A sediment discharge formula, Eq. 9 has been developed, which is a very practical and user-friendly predictive formula to calculate total sediment discharge in alluvial channel for its accuracy, ease of computation and the wide range of applicability. The factor of proportionality, k in Eq. 9 has been checked for a wide range of hydraulic conditions and is a constant with a value of 12.5, irrespective of sediment size, channel aspect ratio, sediment concentration.

The list of the variables tested in the verification of the formula includes: median sediment size from 0.011mm to 28mm, sediment concentration up to 110 kg/m^3, water depth from 0.03 m to 16.4 m, and channel aspect ratio as small as 0.3. The verification exercise for the proposed equation used over 2500 published total-load data from both field and flume studies, and the results showed that, on average, 84% of the data were predicted within a factor of 2, i.e. 0.5 and 2 times of the measured values.

Considering the large database used and the range of applicability of the formula, the result obtained is comparable, if not better than most of the existing total sediment discharge formulas. In particular, the writers have made specific comparisons on the accuracy of Eq. 9 with some of the better-received existing formulas in terms of their predictability for Gilbert's data, the ability to address the effect of aspect ratio and silt transport. Eq. 9 scores remarkably well in all these comparisons. Though no direct comparison with other formulas is presented here, it is obvious that all previous formulas would have used some of the data sets compiled by Brownlie (1981) for verification. In this respect, we can reasonably say that an indirect comparison has been achieved. Our results showed, on average, that 84% of the data sets were predicted within a factor of 2 times of the measured values.

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Table 1. Summary of hydraulic conditions for the data in Fig. 1

Researchers	Sediment size, d_{50} (mm)	Water depth, h (cm)	Channel width, b (cm)	Energy slope Sx1000	No. of data points
Abdel-Aal (1969)	0.105	9-13	30.5	1.7-2.5	10
Brooks. (1957)	0.088, 0.145	4.7-9.1	27.6	1.3-3.3	20
Casey (1935)	1.0, 2.46	3.2-21.9	40.	1.19-5.19	46
Chyn. (1935)	0.59, 0.79, 0.84	5.0-10	61	1.11-3	31
Davies. (1971)	0.15	7.6 - 30	137.2	0.157 - 2.67	33
Einstein & Chien (1955)	0.274, 0.94, 1.3	11 -14	30	12.8 -25.8	14

Gov. of Pakistan (67,68,69)	0.15, 0.25, 0.3	14 - 39	38, 122	0.047-2.6	44
Gibbs & Niell (1972)	4.37	17	122	2.9-5	9
Barton & Lin (1955)	0.18	9 - 42	122	0.6-2.1	28
Mavis et al (1937)	1.41, 1.68, 2.03, 3.73	3 -12	82	1.35-10	106
Range/Total	0.088 - 4.37	3-42	27.6 -137.2	0.047 - 25.8	341

Table 2. Summary of hydraulic conditions in Fig. 3

Researchers	Q(l/s)	b(m)	h(cm)	S*1000	d_{50} (mm)	σ_g	max. c_t(ppm)	Runs
Guy et al (1966)	55-639	2.438	6-40	0.055-14.38	0.19	1.3-2.07	49300	253
Singh (1960)	4-28	0.753	3.8-12	1-14	0.62	1.16	6830	232
Soni (1980)	1.4-9	0.2	2.2-10	2.25-7	0.32	1.3	9200	21
Straub (1954,58)	8-170	0.305,0.914	3.4-24	0.56-7.3	0.19,0.163	1.4	4799	21
Taylor (1971)	47.4	0.851	7-18	0.5-2.05	0.228	1.52	1331	17
Vanoni et al.(1957)	17.4-108.7	0.85	6-17	0.7-2.8	0.137	1.38	3000	13
Vanoni et al.(1965)	4.3-185.5	0.267,1.1	7.3-37	0.45-2.9	0.23,0.206	1.45	1490	14
Willis (1979)	17.48	0.36	11-15	0.83-8.58	0.54	1.12	6669	32
Willis et al.(1972)	78-480	1.219	11-37.5	0.346-2.04	0.1	1.3	19400	96
Ho (1939)	23-63	0.4	12-25	1.02-1.68	2.01	1.99	211	23
Jorrissen (1938)	8.3-36	0.61	2.6-10	1.11-3.33	0.6,0.91	1.8,1.53	1134	13
Kennedy(1961)	5.6-94	0.267,0.851	2-10	1.7-27.2	0.55,0.23	1.14,1.47	58500	41
Kennedy et al. (1965)	39.7	0.851	7-17	0.56-2.5	0.142	1.38	17400	9
Laursen (1958)	24.4-181.9	0.914	7-30	0.43-2.1	0.11	1.2	5150	16
MacDougal (1933)	3.8-36.8	0.61	2-12	1.11-3.33	0.66	1.29	1236	74
Mutter (1971)	11-17	1.219	1.6-5	3.8-7.05	0.26	1.34	10630	7
Nomicos (1957)	8.65-14.4	0.267	7.4-8.6	2-2.5	.091	1.16	8080	30
Nordin (1976)	280-2089	2.38	23.8-85	0.14-5.77	0.25	1.44,1.53	15700	62
O'Brien (1936)	23.3-150	0.914	9-31	0.31-2.73	0.36	1.51	1039	42
Onishi (1972)	24.1-65.3	0.914	7.5-13.5	1.09-2.67	0.25	1.41	3355	14
Pratt (1970)	30.5-105	1.372	7.6-45.7	0.28-2.87	0.478	1.11	560	20
Sato et al.(1958)	50-200	0.78	2-44	0.22-1.63	1.038	1.0	162	206

Table 3. Study on the effect of channel aspect ratio for various predictive formulas based on Williams' data (1970)

	$0.75 \le r \le 1.5$	$0.5 \le r \le 2$	$0.33 \le r \le 3$	Mean value of r
Yang&Lim (with side wall correction)	74.0%	89.8%	94.9%	0.978
Yang&Lim (without side wall correction)	68.3%	88.0%	93.2%	0.885
Yang (1973)	21.5%	48.0%	79%	1.256
Dou (1974)	27.7%	48.0%	75.7%	1.843
Bagnold (1966)	24.8%	45.7%	64.4%	0.916
van Rijn (1984)	22.0%	43.0%	62.1%	1.186

Table 4. Study on the effect of sediment density for various predictive formulas based on US Waterway Experiment Station data (1936)

ρ_s (T/m^3)	Runs	d_{50} (mm)	% Score of predicted sediment discharge and range of discrepancy r											
			$0.75 < r < 1.5$				$0.5 < r < 2$				$0.33 < r < 3$			
			Y*	E-H	VR	K	Y	E-H	VR	K	Y	E-H	VR	K
1.85	26	0.96	50%	50%	61%	15%	69%	76%	85%	23%	92%	88%	92%	65%
1.85	32	0.833	44%	50%	40%	10%	69%	78%	68%	25%	81%	84%	81%	56%
1.74	29	0.833	59%	52%	62%	14%	83%	83%	75%	27%	86%	93%	89%	62%
1.35	31	0.97, 3.107	35%	55%	38%	16%	71%	77%	58%	32%	87%	93%	83%	71%
1.32	64	1.32-3.0	42%	37%	36%	9%	78%	66%	62%	26%	89%	87%	84%	47%
1.26	14	1.16-4	57%	21%	7%	14%	79%	28%	14%	14%	86%	50%	71%	50%
1.11	30	1.29-2.4	60%	10%	50%	3%	73%	33%	83%	10%	93%	50%	93%	27%
1.05	72	0.84-3.2	40%	11%	22%	22%	55%	25%	55%	35%	89%	42%	86%	37%
Total	**298**		**48%**	**36%**	**40%**	**13%**	**72%**	**58%**	**63%**	**24%**	**88%**	**73%**	**85%**	**52%**