NEWTON'S SECOND LAW AND SCOURING

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This paper discusses accepted theories in relation to scour downstream of sills, at bridge piers and abutments. The continuity approach as proposed by Dietz results in an analytical scour equation for clear water scour conditions. Newton's second law, which relates forces acting on a fluid element per unit width, is applied to uniform flow and in the equilibrium phase of the scour process. The depth-averaged dynamic forces and the momentum fluxes acting on a fluid element similar in geometry to a scour hole determine the maximum scour depth downstream of sills, at bridge piers and abutments for clear water scour as well as live bed scour.

Key Words: *Abutment, Bridge pier, Clear water and live bed scour, Newton's second law, Sill, Turbulence*

1. INTRODUCTION

Although the theoretical basis for the design of hydraulic structures is well known, the mechanics of flow and scour has not been well defined and it is not possible to accurately approximate the dimensions of scour holes. This can be ascribed to the complexity of non-uniform flow (= NUF) in the vicinity of hydraulic structures and to the soil and flow interaction.

Recently, Hoffmans (2008) showed that scouring caused by plunging jets can be modelled by a relation which is based on the solution of a set of eight equations with eight unknowns, including Newton's second law. This paper investigates the scour process downstream of sills, at bridge piers and at abutments, by again applying Newton's second law.

2. CONTINUITY APPROACH

CWS (= Clear Water Scour) is defined as scour in which no supply of sediment occurs upstream of the scour hole, thus $q_{s,0} = 0$ or if the depth-averaged flow velocity (U_0) is less than the critical depth-averaged flow velocity (U_c). Applying a rigid lid approach and the equation of continuity in the two-dimensional vertical (2D-V) direction

$$q_{w,0} = q_{w,c} \quad \text{or} \quad U_0 h_0 = U_{c\eta}\left(h_0 + y_{m,e}\right) \qquad (1)$$

the maximum scour depth in the equilibrium phase ($y_{m,e}$) is

$$\frac{y_{m,e}}{h_0} = \frac{U_0 - U_{c\eta}}{U_{c\eta}} = \frac{U_0}{U_{c\eta}} - 1 \qquad (2)$$

in which h_0 is the flow depth upstream of the scour hole, $q_{w,0}$ is the discharge upstream of the scour hole, $q_{w,c}$ is the discharge where the scour hole is at maximum and $U_{c\eta}$ is the critical depth-averaged flow velocity for NUF conditions. Dietz (1969) examined the time-dependent character of scour downstream of a fixed bed protection in which the upstream supply of sediment transport equalled zero ($q_{s,0} = 0$). Based on the 2D-V continuity equation he derived for $y_{m,e}$

$$\frac{y_{m,e}}{h_0} = \frac{\chi_D U_0 - U_c}{U_c} = \frac{\chi_D U_0}{U_c} - 1 \qquad (3)$$

where χ_D (= $U_c/U_{c\eta}$) is a turbulence coefficient that determines the profile of the horizontal flow velocities in the vertical direction. When the near-bed velocities increase or when the bed becomes smoother, χ_D increases. Based on more than 20 flume tests

Dietz showed that χ_D, which varies from 0.78 to 1.09, slightly depends on the bed turbulence or roughness of the fixed bed upstream of the scour hole. Because of the small differences between U_c and $U_{c\eta}$, U_c is assumed to be equal to $U_{c\eta}$ in the further analysis

The definition of the depth-averaged relative turbulence intensity (r_0) is

$$r_0 = \frac{\sqrt{k_0}}{U_0} \quad \text{with} \quad k_0 = \tfrac{1}{h_0}\int_0^{h_0} \tfrac{1}{2}\left(\sigma_u^2 + \sigma_v^2 + \sigma_w^2\right)\mathrm{d}z \qquad (4)$$

where k_0 is the depth-averaged turbulent kinetic energy, and σ_u, σ_v and σ_w are the standard deviations of the fluctuating flow velocities in the x (= longitudinal), y (= transverse) and z (= vertical) directions respecttively.

For uniform flow (= UF), $r_0 = 1.21g^{0.5}/C$ where C is the Chézy coefficient representing the bed roughness and g is the acceleration due to gravity. Typically, for UF, r_0 lies in the range of 0.08 to 0.12 (or $32 < C < 47$ $\mathrm{m}^{1/2}$/s). Applying the definition of the mean bed shear stress, $\tau_0 = \rho(u_*)^2$ with $u_*/U_0 = g^{0.5}/C$, u_* is the bed shear velocity and ρ is the density of water and thus τ_0 can be rewritten as

$$\tau_0 = 0.7\rho\left(r_0 U_0\right)^2 \qquad (5)$$

3. TWO VORTICES IN AN UNIFORM FLOW

In a fluid element in a uniform turbulent flow with dimensions ℓ_x (= $x_2 - x_1$) and ℓ_z in the x and z directions, the following forces are acting: the hydrostatic and dynamic forces, the momentum fluxes and the resistance force near the bed. At the inflow section, $x = x_1$, the horizontal near-bed velocity (u_b) is assumed to be at its minimum, resulting in maximum over pressure. At $x = x_2$, u_b is assumed to be at maximum generating a maximum under pressure. As a result of these assumptions, the flow in the fluid element itself is moving in both the horizontal and vertical directions.

Close to the bed the horizontal flow velocities accelerate from the inflow to the outflow and at the surface they decelerate in the streamwise direction. Using the continuity equation the vertical flow velocities are directed to the bed halfway between the inflow and the outflow of the fluid element at $x = ½(x_1 + x_2)$. Consequently, two rotating eddies with dimensions ℓ_x and ℓ_z occur. One eddy with its centre at $x = x_1$ and $z = ½\ell_z$ rotates clockwise, while the other eddy, with coordinates ($x = x_2$, $z = ½\ell_z$), rotates anticlockwise.

In the x direction the sum of the hydrostatic forces equals zero and is therefore of no importance when simulating erosion and scour. The momentum fluxes M_1 and M_2, are

$$M_1 = \rho q_{w,1} U_1 \quad \text{and} \quad M_2 = \rho q_{w,2} U_2 \qquad (6)$$

are also not interesting, because the flow depths ($h_1 = h_2$), the depth-averaged flow velocities ($U_1 = U_2$) and the discharges ($q_{w,1} = q_{w,2}$) at the inflow and outflow are identical. Hence, the equilibrium of the fluid element is determined by the dynamic forces (load) and the near-bed resistance force (strength).

The instantaneous bed shear stress fluctuates within a large range, resulting in a maximum bed shear stress (τ_m) that is significantly larger than τ_0. Emmerling (1973) found that near the bed the pressure peak (p_m) could reach up to 18 times τ_0. For UF, the shear stress is linear distributed with a maximum close to the bed. If the distributions of the shear stress and the dynamic pressure are similar, the maximum resultant resistance force ($R_{c,m}$) at the bed over ℓ_x is

$$R_{c,m} = \tau_m \ell_x = \tfrac{1}{2} p_m \ell_z - \left(-\tfrac{1}{2} p_m \ell_z\right) \qquad (7)$$

over pressure – (– under pressure)

When τ_m reaches its maximum value, $\tau_m = 18\tau_0$, and assuming that $\ell_x = \ell_z = ½(h_1 + h_2)$, the depth-averaged dynamic pressure (p_0) is

$$p_0 = 9\tau_0 \qquad (8)$$

4. MODELLING OF NON-UNIFORM FLOW

If a fluid element similar in geometry to a scour hole is considered and applying a depth-averaged approach, relevant forces are: M_0 and M_c which read (see also Eq. 6)

$$M_0 = \rho q_{w,0} U_0 \quad \text{and} \quad M_c = \rho q_{w,c} U_c \qquad (9)$$

and the depth-averaged dynamic forces ($P_{dyn,0}$ and $P_{dyn,c}$). Combining Eqs. 5 and 8, $P_{dyn,0}$ at the inflow section is

$$P_{dyn,0} = p_0 h_0 = 6.3\rho\left(r_0 U_0\right)^2 h_0 \qquad (10)$$

For UF, e.g, $r_0 = 0.1$, $P_{dyn,0} = 6.3\rho(0.1U_0)^2 h_0 = 0.06\rho U_0^2 h_0$ or $P_{dyn,0}$ is 6% of M_0. Under such conditions the influence of the dynamic pressure can be neglected with respect to M_0. However, for NUF, e.g. a flow downstream of a sill when $r_0 = 0.2$, $P_{dyn,0} = 6.3\rho(0.2U_0)^2 h_0 = 0.25\rho U_0^2 h_0$ or $P_{dyn,0}$ is 25% of M_0, Hence, the contribution of $P_{dyn,0}$ is significant.

The maximum resultant resistance force ($R_{c,m}$) at the upstream scour slope is related to the near-bed velocity ($u_{b,s}$), which is small with respect to U_0 and is here written as

$$R_{c,m} = 18\tau_m L = 12.6\rho(r_{0,s}u_{b,s})^2 y_{m,e} \cot\Phi \qquad (11)$$

where L is the length on which $R_{c,m}$ is acting and Φ is the angle of repose which lies in the range of 30^o to 45^o. In the recirculation zone the maximum backflow velocity is usually over 20% of U_0 and the mean velocity ($u_{b,s}$) averaged over the upstream scour slope is 5 to 10% of U_0. Assuming that $\Phi = 35^o$, $r_{0,s} = 0.25$ and $y_{m,e} = 2h_0$, $R_{c,m}$ is less than 1% of M_0. Despite the greater bed turbulence at the upstream scour slope with respect to UF, the near-bed velocities are so low that the contribution of $R_{c,m}$ is here not included. Note that for UF, $R_{c,m}$ is not negligible.

Analogous to Eq. 10, where the scour depth is at maximum, $P_{dyn,c}$ is

$$P_{dyn,c} = p_c(y_{m,e} + h_0) = 6.3\rho(r_{0,c}U_c)^2(y_{m,e} + h_0) \qquad (12)$$

Owing to near-bed dynamic pressures which fluctuate in time, bed particles in the scour hole are either lifted up by an under pressure and transported by the mean flow or are pushed into the bed by over pressure and become motionless. Most particles will move with the flow when near-bed dynamic pressures are directed upwards, that is when $P_{dyn,0}$ pushes against the fluid element and when $P_{dyn,c}$ pulls on the fluid element. In this way the scour depth reaches its maximum value.

Applying Newton's second law to a fluid element in a scour hole in which the bed has not been scoured to the point where the scour hole is at its maximum and considering only horizontal forces, the momentum equation reads (Fig. 1)

$$M_0 + P_{dyn,0} = M_c - P_{dyn,c} \qquad (13)$$

5. SCOUR DOWNSTREAM OF SILLS

At the downstream edge of a sill the flow reattaches and a recirculation zone occurs (Fig. 2). In the mixing layer turbulence is generated and this decreases in the relaxation zone. Downstream of the reattachment point, the flow accelerates near the bed

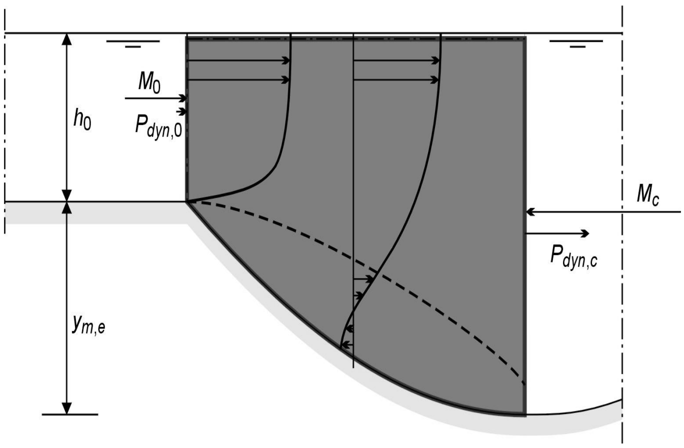

Fig. 1 Forces acting on a fluid element in a scour hole

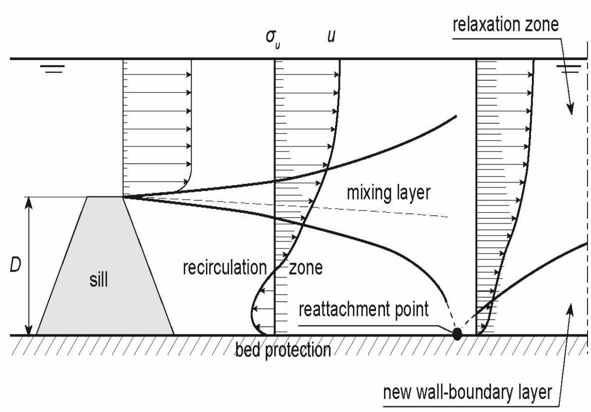

Fig. 2 Flow downstream of a sill

and a new wall-boundary layer develops. Usually the flows downstream of a sill and in a scour hole are similar.

Combining Eqs. 1, 9, 10, 12 and 13, $y_{m,e}$ downstream of a sill for CWS yields

$$\frac{y_{m,e}}{h_0} = \frac{\chi U_0^2 - U_c^2}{U_c^2} \quad \text{with} \quad \chi = \frac{1 + 6.3r_0^2}{1 - 6.3r_{0,c}^2} \qquad (14)$$

in which χ is a turbulence coefficient which varies from 1.7 to 2.5 depending on the turbulence intensities at both the inflow and outflow sections. Figure 3 shows the maximum scour depth relative to the flow depth as function of a dimensionless flow velocity for both normal (r_0 = 0.1) and highly (r_0 = 0.2) turbulent flow conditions.

According to Breusers (1966) $y_{m,e}$ is proportional to the turbulence intensities upstream of the scour depth

$$y_{m,e} \equiv ((1 + 3r_0)U_0 - U_c)^{1.7} \qquad (15)$$

which is identical to $(1 + 6.3r_0^2)U_0^2 - U_c^2$ when $U_0 \geq U_c$ under Live Bed Scour (LBS).

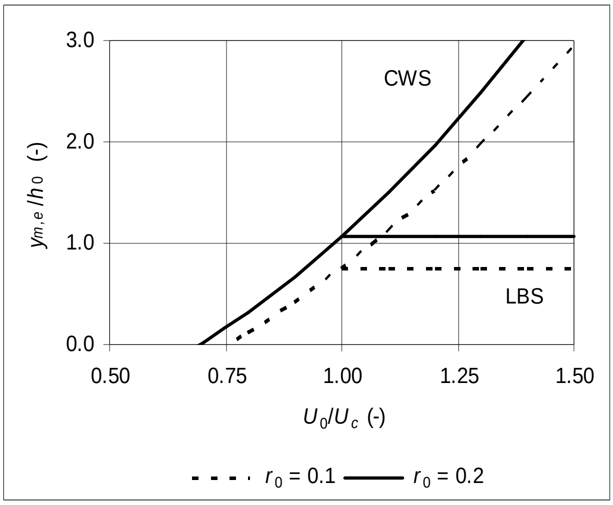

Fig. 3 $y_{m,e}/h_0$ as function of U_0/U_c (with $r_{0,c}$ = 0.25, Eqs. 14&16)

LBS is defined as scour when the sediment transport ($q_{s,0}$) at the inflow section equals the sediment transport ($q_{s,c}$) at the outflow section. For nearly uniform sediment Chabert and Engeldinger (1956) demonstrated that $y_{m,e}$ fluctuates about a mean value in response to the bed passages if $U_0 > U_c$, so $y_{m,e}$ under LBS conditions is

$$\frac{y_{m,e}}{h_0} = \chi - 1 \qquad (16)$$

When $0.08 < r_0 < 0.12$ χ is about 1.75 resulting $y_{m,e}/h_0 \approx 0.75$. The analysis so far shows that if the turbulence upstream of the scour hole increases, the dimensions of the scour hole become larger, which confirms the earlier research activities of Breusers (1966).

Downstream of the storm surge barrier in the Eastern Scheldt where $q_{s,0} = q_{s,c}$, h_0 ranges from 25 m to 40 m, the sill height varies from 15 to 20 m and at the end of the bed protection r_0 lies in the range of 0.15 to 0.25. Using Eq. 16, the calculated $y_{m,e}$ in the tidal river, which consists of fines (diameter of sand is about 200 μm), is nearly equal to the measured $y_{m,e}$ for the three sections considered (Table 1).

6. SCOUR AT SLENDER BRIDGE PIERS

The flow pattern at bridge piers is characterised by the horseshoe vortex combined with the down flow and the wake vortices. Horseshoe vortices are the dominant cause of the scour in front of the pier. The axes of these vortices are horizontal and they erode the foundation of the pier into the shape of a horseshoe. The wake vortices which have vertical axes, arise from flow separation at the sides of the pier.

Table 1 Data of Eastern Scheldt (Hoffmans & Verheij 1997)

	Section R0960	Section R01680	Section H0 620
Experimental data			
length of bed protection	650 m	650 m	650 m
sill height	16.5 m	17.5 m	15.5 m
h_0	40 m	26 m	25 m
$y_{m,e}/h_0$	0.5	1.3	0.8
particle diameter	0.2 mm	0.2 mm	0.2 mm
Computational data			
r_0 [1]	0.16	0.23	0.19
$r_{0,c}$ [= $0.12(y_{m,e}/h_0 + 1)$][1]	0.18	0.28	0.22
χ (Eq. 14)	1.5	2.6	1.8
$y_{m,e}/h_0 = \chi - 1$ (Eq. 16)	0.5	1.6	0.8

[1] (see also Eq. 4 and Hoffmans and Booij 1993)

To examine the influence of turbulence on scouring, Newton's second law on a fluid element with dimensions $\ell_y \ell_z$ at the inflow section (upstream of the scour hole) and $(\ell_y - b)(\ell_z + y_{m,e})$ at the outflow section (where scour hole is at maximum) is applied. (Fig. 4)

$$\begin{aligned} &M_0 \ell_y + p_0 \ell_y \ell_z = \\ &M_c(\ell_y - b) - p_c(\ell_y - b)(y_{m,e} + \ell_z) \end{aligned} \qquad (17)$$

where b is the width of the bridge pier, ℓ_y and ℓ_z are length scales in the y and z direction. Combining Eqs. 1, 9, 10, 12 and 17, $y_{m,e}$ at slender bridge piers for CWS is

$$\frac{y_{m,e}}{b} = \frac{\alpha_{gp}\chi U_0^2 - \alpha_{zp} U_c^2}{U_c^2} \quad \text{with} \quad \alpha_{gp} = \frac{\alpha_{yp}\alpha_{zp}}{\alpha_{yp} - 1} \qquad (18)$$

in which α_{gp}, α_{yp} (= ℓ_y/b) and α_{zp} (= ℓ_z/b) are coefficients which determine the magnitude of the cross sectional areas of the fluid element.

Based on more than 500 LBS experiments, Hoffmans (1995) showed that the proposed relation by Breusers et al (1977)

$$\frac{y_{m,e}}{b} = K_b \qquad (19)$$

gives the best overall score for slender bridge piers with circular form. To calibrate the unknowns in Eq. 18, Eq. 19 is used with K_b = 1.5 for circular forms. K_b is a correction factor for the geometry of the hydrau-

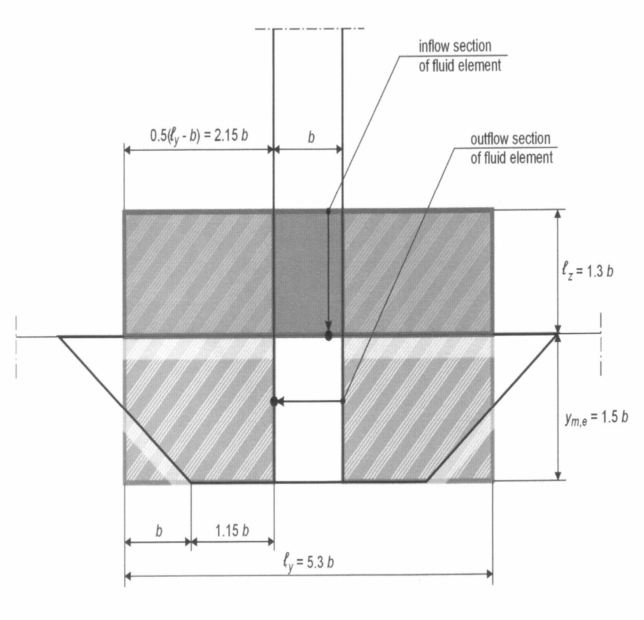

Fig. 4 Fluid element at bridge pier

lic structure or a correction for the turbulence intensity in the scour hole. Upstream of bridge piers the flow is usually in equilibrium. Assuming uniform and hydraulically rough conditions, i.e. $r_0 = 0.10$, the best fit is obtained for $r_{0,c} = 0.25$, $\alpha_{yp} = 5.3$, $\alpha_{zp} = 1.3$.

In the deceleration zone a mixing layer with a horizontal axis occurs and this is incorporated in the fluid element by taking $\alpha_{zp} = 1.3$. The side slope of the scour hole equals the angle of repose and is about 35^o or 1V:1.4H, so the scour-hole width is about 5 times $y_{m,e}$. These dimensions are in agreement with observations.

Analogous to Eq. 16, the maximum scour depth is for LBS (Fig. 5)

$$\frac{y_{m,e}}{b} = 1.6\chi - 1.3 \qquad (20)$$

7. SCOUR AT ABUTMENTS

To analyse the effects of turbulence in relation to load and strength, Eq. 17 is again applied with $b = b' = h_0$ (Fig. 6), resulting in

$$\frac{y_{m,e}}{h_0} = \frac{\alpha_{ga}\chi U_0^2 - U_c^2}{U_c^2} \text{ with } \alpha_{ga} = \frac{\alpha_{ya}\alpha_{za}}{\alpha_{ya} - 1} \qquad (21)$$

in which α_{ga}, α_{ya} (= ℓ_y/h_0) and α_{za} (= ℓ_z/h_0) are coefficients which determine the magnitude of the cross sectional areas of the fluid element.

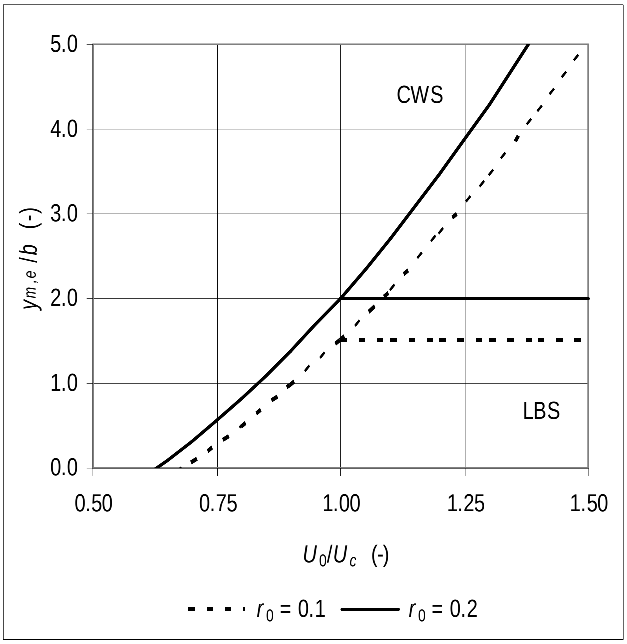

Fig. 5 Scour depth as function of U_0/U_c under turbulent flow conditions using $r_{0,c} = 0.25$, $\alpha_y = 5.3$ and $\alpha_z = 1.3$, (see also Eqs. 18 and 20)

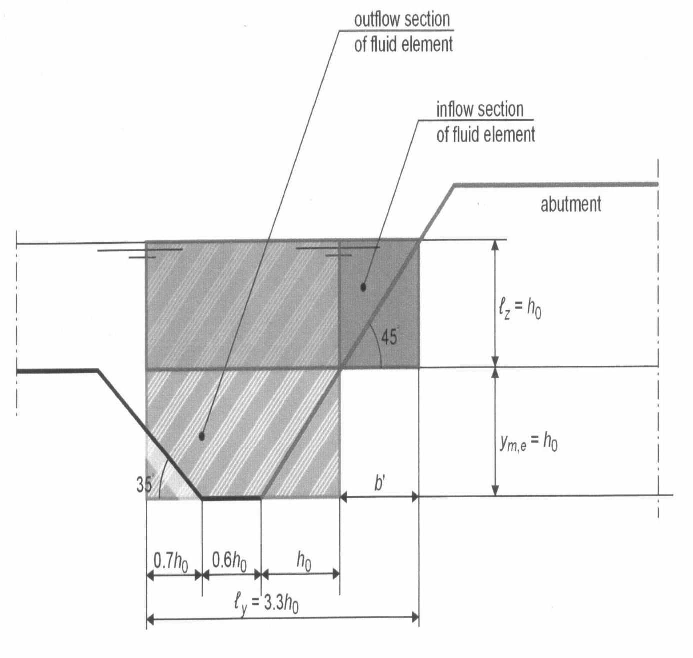

Fig. 6 Fluid element at a streamlined abutment

Recently (Hoffmans 1995), a large number of scour predictors for abutment scour have been compared with experimental data. The result was that a modified Breusers relation extended by adding a term for constriction scour proved to give the best results with respect to live bed scour. If the constriction scour is neglected and if $b > h_0$ this relation for LBS at abutments and bridge piers is (Breusers et al 1977)

$$\frac{y_{m,e}}{h_0} = K_b \qquad (22)$$

where K_b ranges from 0.75 (wing-wall abutment streamlined) to 3.0 (vertical-wall abutment).

To calibrate the unknowns in Eq. 21 the following assumptions have been made: K_b = 1.5 for wing-wall abutments, uniform and hydraulically rough conditions upstream of the scour hole, i.e. r_0 = 0.10 and $r_{0,c}$ = 0.25 giving α_{ya} = 3.3, α_{za} = 1.0. Hence, for LBS $y_{m,e}$ reads

$$\frac{y_{m,e}}{h_0} = 1.4\chi - 1 \qquad (23)$$

In the 3D fluid element a small part of the river in the transverse direction and the total flow depth, i.e. $y_{m,e} + h_0$ are taken into account. When the side slopes of the scour hole equal the angle of repose, with Φ = 35^o, the scour width is about three times the flow depth. Since the distance between the toe of the abutment and the undisturbed streamline is also three times the flow depth, the dimensions of the fluid element are representative for the 3D scour hole.

8. CONCLUSIONS & RECOMMENDATIONS

For both CWS and LBS, scour equations for the maximum depth that are based on accepted theories are derived: Newton's second law and continuity equation. These help us to understand the physical process in local scour holes downstream of sills, at piers and abutments.

It is recommended that experimental data from both flume and prototype tests should be used to validate the proposed scour equations.

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