HORIZONTAL GRANULAR FILTERS

Gijs HOFFMANS[1], Henk den ADEL[2] and Henk VERHEIJ[3]

[1]Member of ISSMGE, Hydraulic Specialist, Deltares
(P.O. Box 177, 2600 MH Delft, The Netherlands)
E-mail:gijs.hoffmans@deltares.nl
[2]Consultant Experimental Research, Deltares
E-mail:henk.denadel@deltares.nl
[3]Lecturer, Delft University of Technology & Deltares
E-mail:henk.verheij@deltares.nl

This study investigates the stability of horizontal granular filters which protect the underlying soil, i.e., the base layer, from erosion by static and fluctuating loads. The head usually characterizes the static load over hydraulic structures, whereas the fluctuating load represents turbulence caused by the geometry of hydraulic structures or by the roughness of the top layer. The erosion resistance or strength of granular filters is mainly characterised by the geometrical properties of the materials used. In general, two types of granular filters can be distinguished, namely geometrically closed and geometrically open filters. Here filter equations based on accepted theories are discussed for laminar and turbulent flow in horizontal filters. When the top layer is influenced by bed turbulence generated in open channel flow, filter equations are also derived.

Key Words: *Granular Filters, Laminar Flow, Open Channel Flow, Turbulence, Turbulent Flow*

1. INTRODUCTION

Typically, granular filter elements (stone, gravel and sand) are robust and give a good contact interface between the filter and base layers. Granular filters can smoothen bed irregularities and thus provide a more uniform construction base; moreover, they are easy to repair and sometimes they may be self-healing. The major disadvantage of granular filters is the difficulty of achieving uniform construction underwater to ensure the required thickness of the filter layers.

Granular filters protect the underlying soil, i.e. the base layer, from erosion by the static or mean hydraulic load and fluctuating loads. The head usually determines the static load over hydraulic structures, while the fluctuating load represents turbulence caused by the geometry of hydraulic structures or by the roughness of the top layer.

The erosion resistance or strength of granular filters is mainly characterised by the geometrical properties of the materials used. In a geometrically closed filter, the ratio between the largest and smallest particles is so small that the bigger particles block the smaller ones. Geometrically open filters are characterised by particles that can erode through the filter layer.

Equations used to design and test horizontal filters which are influenced by laminar or turbulent flow are discussed. Moreover, this study also examines the decrease of bed turbulence in filter layers.

2. HYDRAULIC MODELLING

To model static and fluctuating hydraulic loads various parameters can be used, for example, energy slope (or filter velocity), shear velocity (or shear stress), pressure fluctuations (or turbulent kinetic energy) and drag, lift and shear forces. The energy slope, shear velocity and pressure fluctuations can all be related to forces acting on particles.

In open channel flow turbulence is generated close to the bed and in non-uniform flow turbulence is also caused by the geometry of hydraulic structures. The blunter the hydraulic structure and the rougher the bed, the higher the bed turbulence. In granular filters water flows through open spaces and when the flow reattaches, small mixing layers occur, generating turbulence. The vortices in these open spaces are much smaller than the vortices in open channel flow and thus contain less energy. The depth-averaged relative turbulence intensity (r_0) in channel flow and the mean relative turbulence intensity ($r_{0,f}$) in the filter layer are defined as

$$r_0 = \sqrt{k_0}/U_0 \quad \text{and} \quad r_{0,f} = \sqrt{k_{0,f}}/u_f \quad \text{with}$$
$$k_0 = \frac{1}{h}\int_0^h \tfrac{1}{2}\left(\sigma_u^2 + \sigma_v^2 + \sigma_w^2\right)\mathrm{d}z \tag{1}$$

where h is the flow depth, k_0 is the depth-averaged turbulent kinetic energy in open channel flow, $k_{0,f}$ is the mean turbulent kinetic energy in the filter layer, u_f is the filter velocity, U_0 is the depth-averaged flow velocity in open channel flow and σ_u, σ_v and σ_w are the standard deviations of the fluctuating flow velocities in the x (= longitudinal), y (= transverse) and z (= vertical) direction respectively. For uniform flow $r_0 = 1.21g^{0.5}/C$ where C is the Chézy coefficient and g is the acceleration due to gravity.

To determine u_f and its critical value ($u_{c,f}$), the Forchheimer equation and an equation similar to Chézy are applied. When granular filters are influenced by bed turbulence from channel flow the load is described using the shear stress approach of Grass (1970). The decrease of the load in the filter layer is modelled by an exponential function.

Forchheimer found a relation between the mean energy slope (S) and u_f that is non-linear at sufficiently high velocities. This non-linearity increases with u_f and is caused by turbulent effects of the flow in the filter

$$S = au_f + bu_f^2 \tag{2}$$

in which a [s/m] and b [s^2/m^2] are dimensional coefficients. The Forchheimer equation assumes that Darcy's law is still valid. However, an additional term is added to account for the increased S. Based on permeability measurements, Den Adel (1986) found for the coefficients a and b

$$a = 160\frac{\nu}{g}\frac{\left(1-n_f\right)^2}{n_f^3 d_{f15}^2} \quad \text{and} \quad b = \frac{2.2}{gn_f^2 d_{f15}} \tag{3}$$

where d_{f15} is the particle (or grain) diameter in the filter layer for which 15% of the particles is finer than d_{f15}, n_f is the porosity of the filter and ν is the kinematic viscosity. Applying a and b, the predictability of S in Eq. 2 lies in the range of $⅓ < \zeta < 3$, where ζ is the ratio of the measured and calculated S.

3. INCIPIENT MOTION

Particle transport occurs when there is no balance between load and strength. When the load is less than some critical value, particles remain motionless and can be considered as fully stable. But when load exceeds its critical value, particle motion begins. The initiation of motion is difficult to define. This can mainly be ascribed to phenomena that are random in time and space.

When dealing with particle stability in granular filters, the exact shape of the distribution of both load and strength are irrelevant because a characteristic load[1] ($\tau_{0,k}$) and a characteristic strength[1] ($\tau_{G,k}$) can be defined. A characteristic value is a value that is higher or lower than the mean value. Usually characteristic values are expressed as a mean value and a fraction of the standard deviation. Consequently, the problem of particle stability could be transferred to the magnitude of this fluctuation. Using the hypothesis of Grass (1970), which is based on statistical assumptions for both $\tau_{0,k}$ and $\tau_{G,k}$ read (Fig. 1)

$$\text{characteristic load:} \quad \tau_{0,k} = \tau_0 + \gamma\sigma_0 \tag{4}$$
$$\text{characteristic strength:} \quad \tau_{G,k} = \tau_G - \gamma\sigma_G \tag{5}$$

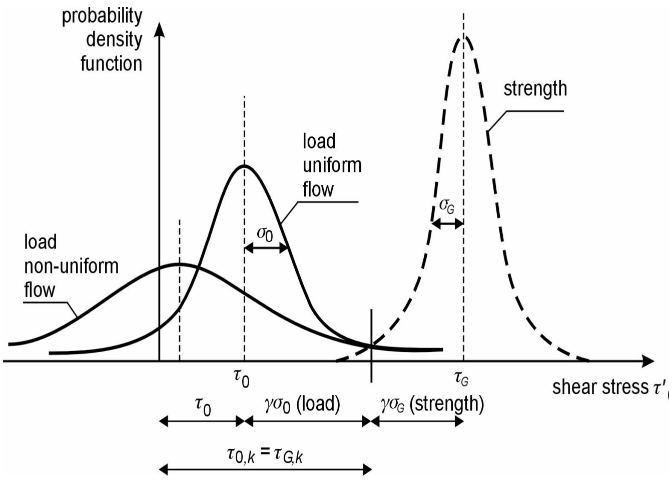

Fig. 1 Probability functions of load and strength (Grass 1970)

where γ is determined by an allowable transport of the bed material, σ_0 is the standard deviation of $\tau_{0,k}$, σ_G is the standard deviation of $\tau_{G,k}$, τ_0 is the mean load (or mean bed shear stress) and τ_G is the mean strength (or critical mean bed shear stress) as given by Grass.

If $\tau_{0,k} = \tau_{G,k}$ and $\sigma_G = V_G\,\tau_G$ with $\tau_G = \Psi_G\,\Delta\,\rho\,g\,d_{50}$ (analogous to Shields), a general equation for the filter layer (or top layer) follows

[1] Under channel flow, $\tau_{0,k}$ and $\tau_{G,k}$ are the instantaneous bed shear stress and the critical instantaneous bed shear stress.

$$\Delta_f d_{f50} = \frac{\tau_0 + \gamma\sigma_0}{\Psi_{Gf}\rho g(1 - V_{Gf}\gamma)} \qquad (6)$$

where d_{f50} is the mean particle diameter of the filter layer, V_{Gf} is the variation coefficient that represents the influence of the non uniformity of the filter layer, Δ_f (= ρ_s/ρ – 1) is the relative density of the filter material, ρ is the density of the water, ρ_s is the density of the filter material and Ψ_{Gf} is related to the critical Shields parameter (Ψ_c, Fig. 2).

A specific transport will occur if $\tau_{0,k} = \tau_{G,k}$. For uniform flow, $\sigma_0 \approx 0.4\tau_0$, Grass found that a bed of nearly uniform sand, $V_{Gf} \approx 0.3$, was completely stable for $\gamma = 1$, while for $\gamma = 0$ a significant transport of sediment particles was observed. Based on his tests, he reported that for $\gamma = 0.625$ and using $\Psi_{Gf} \approx 1.5\Psi_c$, the criterion of Shields was met for the initial movement of sands up to a size of 250 μm.

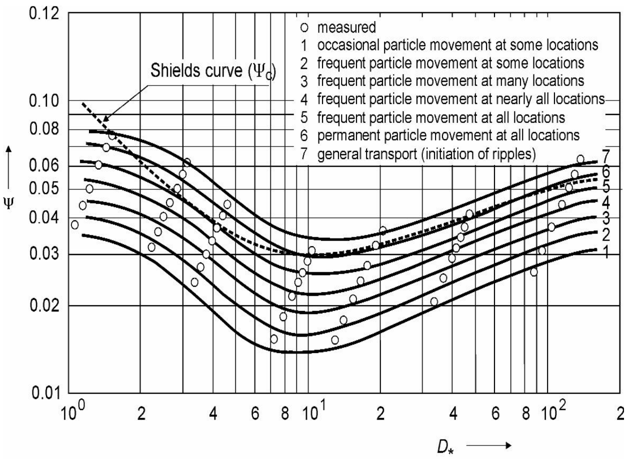

Fig. 2 Shields diagram Ψ as function of D_* (= $d(\Delta g/\nu^2)^{0.33}$)

4. HORIZONTAL FILTERS WITHOUT BED TURBULENCE

When the water flows parallel to the interface, the gradient in both layers is about the same, causing u_f in the filter layer to be much higher than in the base layer, because of the greater hydraulic conductivity. At the interface there will be a velocity gradient, inducing a shear stress at the upper fines in the base layer. Van der Meulen (1984), Klein Breteler (1989) and Broekens (1991) conducted flume experiments in which the flow was parallel to the filter and base layers. In these tests the flow was laminar as well as turbulent and no open channel flow above the filter was considered.

The critical filter velocity ($u_{c,f}$) is a function of filter characteristics on the one hand and the critical bed shear velocity ($u_{*c,bf}$) at the interface of filter and base layer on the other hand. Analogous to channel flow, i.e., using Chézy's equation, $u_{c,f}$ could be written as

$$u_{c,f} = \frac{C_f}{\sqrt{g}} u_{*c,bf} \text{ with } C_f = \alpha_{15}\sqrt{g}\left(\frac{d_{f15}}{d_{b50}}\right)^{1/6} \text{ and}$$

$$u_{*c,bf} = \sqrt{\Psi_{c,b}\Delta_b g d_{b50}} \qquad (7)$$

where C_f is a coefficient [$m^{0.5}/s$] representing the resistance in the filter layer, α_{15} is a coefficient, Δ_b is the relative density related to the base layer and $\Psi_{c,b}$ is the critical Shields parameter related to d_{b50}. Combining Eqs. 2, 3 and 7 and considering laminar flow (thus $Re_f = d_{f15}u_f/\nu < 1000$ and $b = 0$ in Eq. 2) yields (Fig. 3)

$$S_{c,lam} = \alpha_L \frac{(1-n_f)^2 \nu\sqrt{\Delta_b}}{n_f^3\sqrt{g}} \frac{(d_{b50})^{1/3}}{(d_{f15})^{1 5/6}} \text{ with}$$

$$\alpha_L = 160\alpha_{15}\sqrt{\Psi_{c,b,lam}} \approx 65 \qquad (8)$$

Substitution of Eq. 7 in Eq. 2 with $a = 0$, Eq. 2 for turbulent flow reads, thus $Re_f > 1000$ (Fig. 4)

$$S_{c,tur} = \alpha_T \frac{\Delta_b}{n_f^2}\left(\frac{d_{b50}}{d_{f15}}\right)^{2/3} \text{ with}$$

$$\alpha_T = 2.2\alpha_{15}^2\Psi_{c,b,tur} \approx 0.1 \qquad (9)$$

The Shields diagram shows that for laminar flow or for fines smaller than 0.1 mm, $\Psi_{c,b,lam}$ could reach values up to 0.1. Assuming that $\Psi_{c,b,lam} = 0.1$ and using Eq. 8, $\alpha_{15} = 65/(160 \cdot 0.1^{0.5}) = 1.28$. Substitution of $\alpha_{15} = 1.28$ in Eq. 9 gives, $\Psi_{c,b,tur} = 0.1/(2.2 \cdot 1.28^2) = 0.03$, which is in agreement with turbulent flow tests.

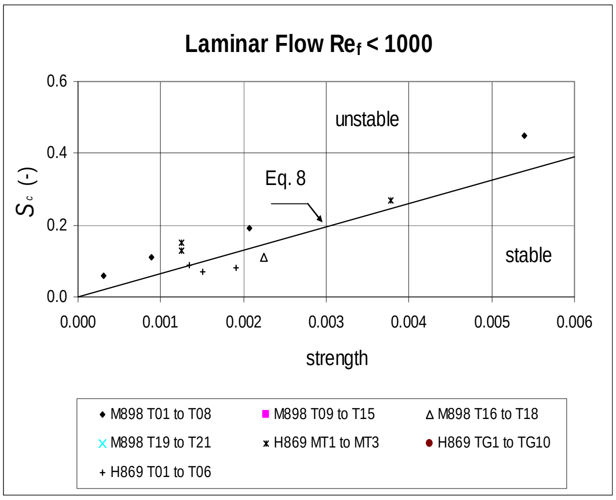

Fig. 3 S_c as function of strength: $\frac{(1-n_f)^2 \nu\sqrt{\Delta_b}}{n_f^3\sqrt{g}} \frac{(d_{b50})^{1/3}}{(d_{f15})^{1 5/6}}$

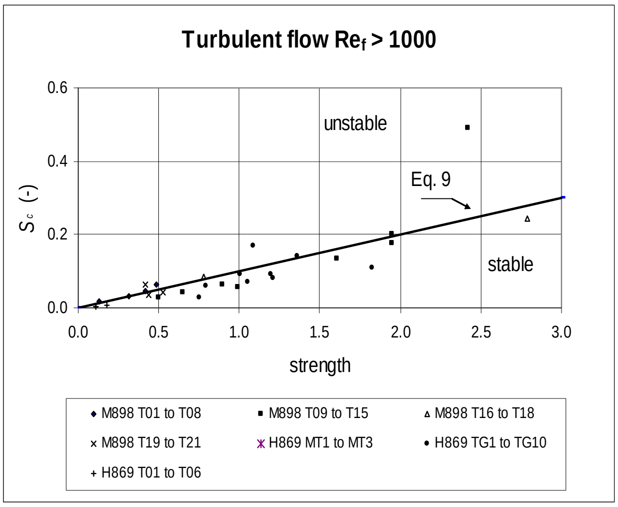

Fig. 4 S_c as function of strength: $\frac{\Delta_b}{n_f^2}\left(\frac{d_{b50}}{d_{f15}}\right)^{2/3}$

According to Aguirre Pe (Hoffmans 2006), who investigated the incipient motion of gravel (d_{f50} = 0.05 m) under steep channel flow conditions ($0.01 < S_c < 0.1$), r_0 lies in the range of 0.3 to 0.6. In the tests carried out by Deltares, d_{f15}/d_{b50} varied from 30 to 300 giving $7 < C_f < 10$ and $0.4 < r_{0,f}$ ($= 1.21g^{0.5}/C_f$) < 0.5, so the order of magnitudes of $r_{0,f}$ and r_0 are the same, or $O(r_{0,f}) = O(r_0)$.

As given by Koenders (1985), who used an entirely different approach to solving the equilibrium of particles in granular filters, in the low and high gradient limits S_c is proportional to

$$S_{c,lam} \equiv \frac{(d_{b50})^{2/3}}{(d_{f15})^{5/3}} \quad \text{and} \quad S_{c,tur} \equiv \frac{d_{b50}}{(d_{f15})^{1/3}} \qquad (10)$$

whence follows that Koenders results show similar proportions to those in the proposed Eqs. 8 and 9.

5. HORIZONTAL FILTERS INFLUENCED BY BED TURBULENCE

Hoffmans et al. (2000) discussed load in a horizontal one-layer filter with a thickness (D_F) above the base material in open channel flow (Fig. 5). Equations for granular filters based on the Navier Stokes equation for uniform flow, Forchheimer's equation and the hypothesis of Boussinesq are deduced and validated.

The distribution of τ defined as load in a one-layered filter, is

$$\tau(z) = \tau_{bf} e^{-\xi z} + \tau_0 e^{\xi(z-D_F)} \quad \text{with}$$

$$\xi = \sqrt{\frac{2gb}{\alpha_v d_{f15}}} \approx \frac{5.5}{d_{f15}} \qquad (11)$$

where ξ is a damping parameter and τ_{bf} is the mean load at the interface of the filter-base layer. Shimizu et al (1990) found for the damping $O(\xi D_F) = 1$. As in the case of channel flow, τ_{bf} could be related to τ_0 as follows (see also Appendix A)

$$\tau_{bf} = \eta\tau_0 \quad \text{with} \quad \eta = 0.7 r_{0,f}^2 \qquad (12)$$

If either τ_0 or $r_{0,f}$ increases, τ_{bf} also increases which is in agreement with observations.

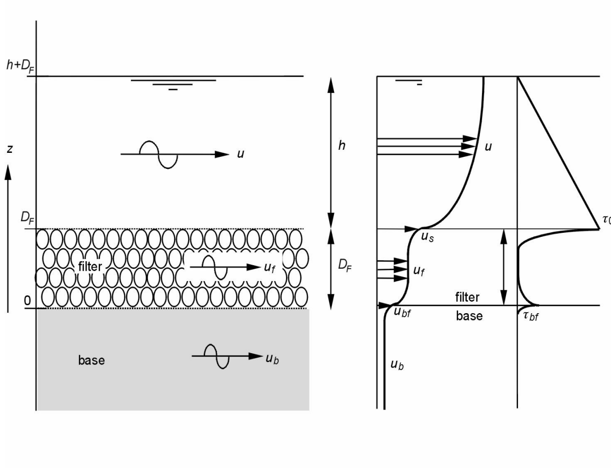

Fig. 5 Overview of definitions for a one-layer filter

In a similar way, equations can be derived for filters at the interface of the filter-base layer. Applying the hypothesis of Grass, the characteristic load ($\tau_{bf,k}$) and the characteristic strength ($\tau_{c,bf,k}$) of the base material at $z = 0$ are

$$\tau_{bf,k} = \eta(\tau_0 + \gamma\sigma_0) \qquad (13)$$

$$\tau_{c,bf,k} = \tau_{Gb} - \gamma\sigma_{cb} = \Psi_{Gb}\Delta_b \rho g d_{b50}(1-\gamma V_{Gb}) \qquad (14)$$

Combining Eqs. (6), (13) and (14) for geometrically open filters gives

$$\frac{d_{f50}}{d_{b50}} = \frac{1}{\eta}\frac{1-\gamma V_{Gb}}{1-\gamma V_{Gf}}\frac{\Psi_{Gb}}{\Psi_{Gf}}\frac{\Delta_b}{\Delta_f} \qquad (15)$$

With Eq. (15) the influence of particle gradation on the stability of the base material can be qualitatively explained. For example, when the base material is more graded than the filter material, thus $V_{Gb} > V_{Gf}$, the required ratio d_f/d_b is less than the value in situations where base and filter materials do have the same gradation. If only the filter material is broadly graded, thus $V_{Gb} < V_{Gf}$, the maximum value of d_f/d_b is higher than for similarly graded materials. These predictions correspond with observations in flume tests.

A widely graded base material has more fines than a more uniform material. The material in the filter layer has to prevent the erosion of the fines. This can only be achieved by reducing u_f or by putting more fines into the filter layers. A widely graded material in the filter layer has relatively more fines, which reduces u_f and so τ_{bf}. Hence, the widely graded filter material can have a d_{f50} that is larger than for uniform material.

Van Huijstee and Verheij (1991) conducted tests where bed turbulence was generated under uniform flow. In addition, a distinction was made between simultaneous instability of the base and top layers and instability of either the top or base layer. In all these tests D_F / d_{f50} varied from 1.5 to 4.5 and the critical d_{f50}/d_{b50} obtained from tests ranged from 40 to 400. Figure 6 shows the critical $[d_{f50}/d_{b50}]_{measured}$ when erosion occurs versus the critical $[d_{f50}/d_{b50}]_{computed}$ assuming that $\eta = 0.7(r_0)^2$ (or $r_{0,f} = r_0$ see also section 4), $V_{Gb} = V_{Gf}$ and $\Delta_b = \Delta_f$

$$\left[\frac{d_{f50}}{d_{b50}}\right]_{computed} = \frac{1}{0.7 r_0^2}\frac{\Psi_{cb}}{\Psi_{cf}} \quad \text{with}$$

$$r_0 = 1.21\kappa\left(\ln\frac{6R_h}{d_{f50}}\right)^{-1} \qquad (16)$$

where R_h is the hydraulic radius and κ (= 0.4) is the constant of Von Kármán. Since for most of the tests a thin filter layer was used, 100% of the measurements lie in the range of $0.5 < \zeta < 2$, where ζ is the ratio between the measured and computed d_{f50}/d_{b50}.

For uniform equilibrium and non-uniform gradually varied flows, r_0 ranges typically from $r_0 = 0.042$ (or $\eta = 0.0013$ for a large smooth channel) up to $r_0 = 0.126$ (or $\eta = 0.0113$ for a small rough channel). For steep channel flow and non-uniform flow when $0.2 < r_0 < 0.5$, η lies in the range of 0.028 to 0.18. Hence, for very high turbulence intensities, say $r_0 > 0.25$ geometrically closed filters are required with $d_{f50}/d_{b50} < 10$ or $\eta > 0.10$.

Bezuijen and Köhler (1998) examined the stability of revetment structures, which is governed by the interaction between pore water on the one hand, and the top layer, filter layer and base layer on the other hand. Based on theoretical considerations they deduced an exponential equation for the pressure decrease similar to Eq. 11, which here is expressed in terms of mean relative turbulence intensities with χ as a coefficient as

$$r_{0,f}^2 = r_0^2 \exp\left(-\frac{\chi D_F}{d_{f15}}\right) \qquad (17)$$

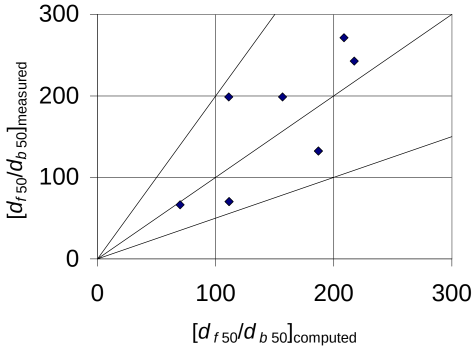

Fig. 6 $[d_{f50}/d_{b50}]_{measured}$ versus $[d_{f50}/d_{b50}]_{computed}$ Experiments in which simultaneous erosion occurs in filter and base layer (Van Huijstee and Verheij 1991)

Hence, $r_{0,f}$ depends not only on r_0, but also on D_F and d_{f15}. Combining Eqs. 15 and 17 gives

$$\frac{D_F}{d_{f15}} = \chi^{-1}\ln\left(0.7 r_0^2 \frac{d_{f50}}{d_{b50}}\frac{1-\gamma V_{Gf}}{1-\gamma V_{Gb}}\frac{\Psi_{cf}}{\Psi_{cb}}\frac{\Delta_f}{\Delta_b}\right) \approx$$

$$\chi^{-1}\ln\left(0.7 r_0^2 \frac{d_{f50}}{d_{b50}}\right) \qquad (18)$$

Wörman (1989) investigated granular filters at bridge piers. Based on accepted theories he arrived at the following relation for one-single layer bed

$$D_F = 0.16\frac{\Delta_f}{\Delta_b}\frac{n_f}{1-n_f}\frac{d_{f85}d_{f15}}{d_{b85}} \qquad (19)$$

For nearly uniform-graded materials when $d_{b85}/d_{b50} = d_{f50}/d_{f15} \approx 1.25$, $n_f = 0.4$, and $\Delta_b = \Delta_f$, Eq. 19 can be rewritten as

$$\frac{d_{f50}}{d_{b50}} = 11.7\frac{D_F}{d_{f50}} \qquad (20)$$

Figure 7 shows Wörman's equation (Eq. 20), Eq. 18 as an envelop curve using $\chi = 0.3$ and $r_0 = 0.25$ as first approximations and experimental data from Van Huijstee and Verheij (1991), which all lie in the unstable part of the diagram. The interesting section for designing and testing geometrically open filters in non-uniform flow is the stable part that lies above Wörman's equation or above Eq. 18 and adjacent to the zone representing geometrically closed filters.

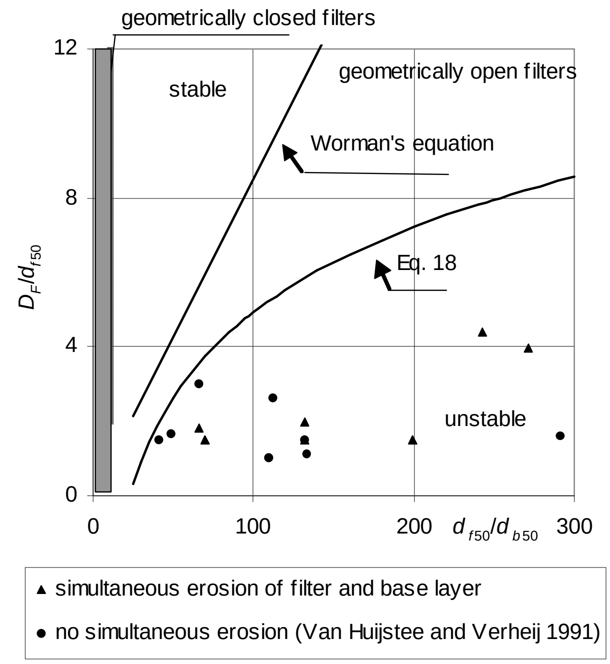

Fig. 7 D_F/d_{f50} versus the critical $[d_{f50}/d_{b50}]$. In Eq. 18 χ = 0.3 and r_0 = 0.25 are first estimations.

6. CONCLUSIONS

For horizontal geometrically open filters without bed turbulence, equations that are based on the Forchheimer and Chézy equations have been deduced and validated using flume experiments. The best guess predictors, Eqs. 8 and 9, are valid for laminar and turbulent flow respectively.

For horizontal geometrically open filters in which open channel flow is considered, a filter equation which is based on the shear stress approach, as proposed by Grass, is derived. The influence of both the thickness of the filter layer (Eq. 17) and grading effects of the filter and base materials has been shown qualitatively. Although Eqs. 15 and 18 were validated by using uniform flow tests, no validation has been carried out for non-uniform flow conditions.

Wörman's equation is validated applying data from filter layers around bridge piers where the flow is highly turbulent. However, the horseshoe vortices and the Kármán vortex streets are not representative for all types of non-uniform flow.

APPENDIX A

The mean load at the interface of the filter-base layer and the mean bed shear stress are defined as

$$\tau_{bf} = \rho u_{*bf}^2 \quad \text{and} \quad \tau_0 = \rho u_*^2 \qquad \text{(A1)}$$

For open channel flow, Chézy's equation reads

$$u_* = U_0 \sqrt{g} / C = 0.83\, r_0 U_0 \qquad \text{(A2)}$$

In the present study, Eq. A2 is used to model the resistance in the filter layer as

$$u_{*bf} = u_f \sqrt{g} / C_f = 0.83 r_{0,f} u_f \qquad \text{(A3)}$$

Substitution of Eqs. A2 and A3 in A1 gives

$$\tau_{bf} = 0.7\rho\left(r_{0,f} u_f\right)^2 \text{ and } \tau_0 = 0.7\rho\left(r_0 U_0\right)^2 \qquad \text{(A4)}$$

Assuming that $u_* = u_f$, η is

$$\eta = \tau_{bf} / \tau_0 = 0.7 r_{0,f}^2 \qquad \text{(A5)}$$

REFERENCES

1. Adel, den, H.: Analysis of permeability measurements using Forchheimer's equation, Report CO 272550/6 (in Dutch), Deltares, Delft, 1986.
2. Bezuijen, A., and Köhler, H.J.: Filter and revetment design of water imposed embankments induced by wave and draw-down loadings. in: Dikes and Revetments, Edt. K.W. Pilarczyk, Balkema, Rotterdam, 1998.
3. Broekens, R.D.: Transport of base material in granular filter, influence of side slopeson filter material (in Dutch), Report H869, Deltares, Delft, 1991.
4. Grass, A.J.: Initial instability of fine bed sand, *Hydraulic Division, Proc. of ASCE*, Vol. 96, HY3, pp. 619-632, 1970.
5. Hoffmans, G.J.C.M., Adel, den, H., Verheij, H.: Shear stress concept in granular filters, Proc. of an Int. Symp. Organised by TC 33 of ISSMGE, Melbourne, 2000.
6. Hoffmans, G.J.C.M.: Stability of stones under non-uniform flow, Rep. DWW-2006-085, Rijkswaterstaat, Delft, 2006.
7. Huijstee, van, J.J.A., and Verheij, H.J.: Design relations for granular filters in bed protections (in Dutch), Report Q572, Deltares, Delft, 1991.
8. Klein Breteler, M.: Sand transport in granular filters, horizontal and steady flow (in Dutch), Report H869, Deltares, Delft, 1989.
9. Koenders, M.A.: Hydraulic criteria for filters, Estuary Physics, Physical modelling consultants, Surrey, UK, 1985.
10. Shimizu, Y., Tsujimoto, T. and Nakagawa, H.: Experiment and macroscopic modelling of flow in highly permeable porous medium under free surface flow, *J. of Hydroscience and Hydraulic Engineering*, Vol. 8, No. 1, pp. 69-78, 1990.
11. Meulen, van der T.: Granular filters, Flow parallel to the interface (in Dutch), Report M898-IV, Deltares, Delft, 1984.
12. Wörman, A.: Riprap protection without filters, *J. Hydraulic Engineering*, Vol. 115, No. 12, pp.1615-1630, 1989.