DISCRETE ELEMENT MODELLING OF SOIL EROSION

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ABSTRACT: The process of internal erosion of fine particles from a soil has been modelled using two-dimensional discrete element analysis of assemblies of circular discs of various gradings. Maintaining a sample under constant stresses, finer particles have been progressively removed and the resulting deformations observed. The stability of the particle removal is related to the stress ratio which the sample is experiencing. The material is described by a model in which strength is controlled by state parameter - distance from the critical state line. As the grading is narrowed the critical state line rises, but as particles are removed the specific volume rises more rapidly and the material feels looser.

Keywords: *erosion, discrete element modelling, stress:strain response, critical states, grading*

1 INTRODUCTION

When water flows through a broadly graded natural or man-made soil there is a possibility of erosion of the smaller particles within the soil leading to a narrowing of the grading. Of course rules have been developed for the design of filters in order to try to avoid the occurrence of such internal erosion but it is of interest to explore the mechanical consequences of its occurrence. Numerical studies of two dimensional assemblies of circular discs have been performed in order to understand more about the way in which such internal erosion takes place and the effect that it might have on subsequent mechanical response. These studies form part of a more general study of the modelling of effects of changing grading of soils: internal erosion is a process which removes smaller particles and narrows the grading; particle breakage under load is a process which creates more smaller particles and broadens the grading. Particle breakage is driven by evolution laws which link the probability of particle breakage with the stresses and other aspects of the current state of the soil. The evolution of grading accompanying internal erosion is influenced by the stress conditions and more importantly by the flow regime. The seepage velocity must be high enough to remove and transport the particles and the downstream structure of the soil must in turn be able to receive the particles that are thus transported. In either case the change of the material while it is being studied adds a further dimension to the definition of the state of the material which cannot be ignored.

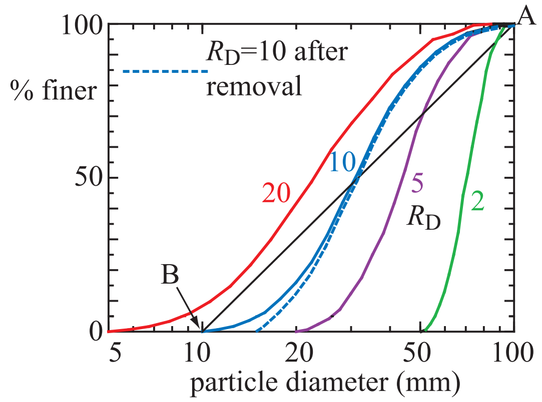

Fig. 1. Grading characterised by ratio of maximum and minimum particle sizes R_D

2 DISCRETE ELEMENT MODELLING OF SOIL EROSION

The current grading of a soil can be characterised in various different ways. There is some advantage in using a grading state index which has limiting values of 0 and 1: the lower value corresponding to a soil which contains particles only of one single size and the higher value corresponding to a soil which has a limiting grading possibly of a self-similar fractal nature ([1], [2], [3]). However, for the purposes of the present study we will characterise the grading using the ratio of maximum and minimum particle sizes $R_D = d_{max}/d_{min}$ (Fig 1).

Samples of circular discs have been prepared with gradings described by $R_D = 2$, 5, 10, 20, with $d_{max} = 100$mm in all tests. The test specimens had an initial size of 750×1500mm. All analyses have been performed using a simple linear elastic limiting frictional contact model with the limiting friction between the discs set at 0.25. However, some control over the initial density of packing of the material can be obtained by using a different interparticle friction during sample preparation [4].

Specimens were prepared under zero gravity. First, discs with interparticle friction coefficient μ_g were generated such that the porosity was equal to a specified average porosity n_g. The required number of discs for the chosen size distribution were placed randomly in the container. With high porosity $n_g = 0.40$, there are no contact points and the initial packing is loose; with low porosity, the particles would initially overlap, $n_g = 0.10$, and

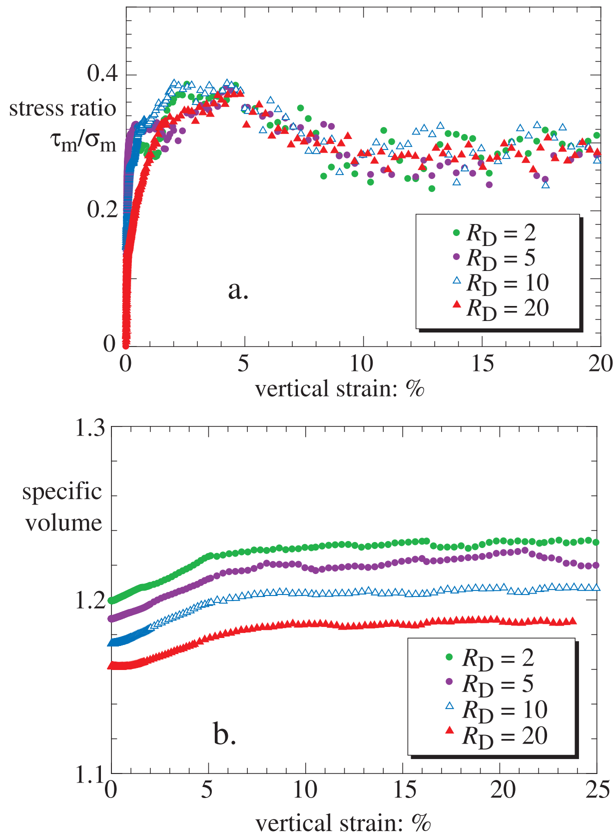

Fig. 2. Tests with constant grading: (a) mobilised friction; (b) specific volume.

the initial packing is dense. Overlapping particles are subject immediately to movement as out-of-balance contact forces equilibrate. The specimen in its container was then subjected to a steadily increasing isotropic plane strain stress by slowly moving the walls of the container inwards or outwards. The initial isotropic stress was regulated to the required value of mean normal stress $\sigma_{m0} = k_n \times 10^{-4} = 50$kPa. The interparticle friction coefficient was then changed instantaneously from μ_g to 0.25 and equilibrium under the applied isotropic stress was re-established. The resulting assembly of particles had an initial void ratio e_0 corresponding to the stress σ_{m0}. Stresses were then applied in addition to the prescribed compression stress in order to shear the sample. The interparticle friction was kept constant at 0.25 throughout the shear tests.

3 MONOTONIC SHEARING

A first series of biaxial compression tests was performed on loose and dense samples with different initial gradings, R_D, by steadily driving downwards one rigid boundary of the specimen container while adjusting the other

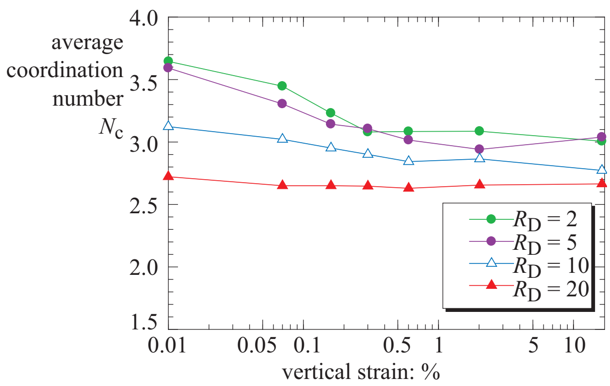

Fig. 3. Tests with constant grading: average coordination number.

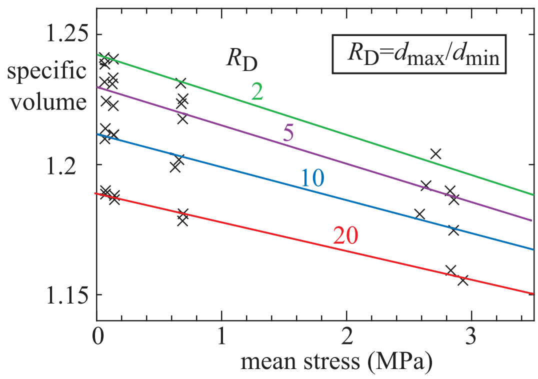

Fig. 4. Tests with constant grading: critical state lines in compression plane.

boundary to maintain constant mean plane strain stress ($s = (\sigma_v + \sigma_h)/2$) in the sample. The results of typical tests are shown in Figs 2, 3. The eventual mobilised friction at large strains is somewhat independent of the initial grading and density - an indication that a critical state condition has been reached in these tests. It is important to note that these simulations show that not only the density and shear stress (or mobilised friction) reach a steady, asymptotic state as shearing continues, but also the particle arrangement as typified by the average coordination number reaches a steady condition, albeit at a somewhat larger strain. (The average coordination number N_c indicates the average number of contacts per particle and gives an indication of the overall stability of the fabric.) This is a necessary confirmation that an asymptotic critical state has actually been attained. A similar result has been found in analyses of two-dimensional assemblies of angular particles [5] for which the strain needed to reach a stable geometric fabric can be greater than 100% depending on the relative orientations of deposition direction and major principal stress.

As the grading becomes broader the particles pack more efficiently and the available range of stable void ratios of the material falls. The critical state conditions in Fig 4 show the lowering of the critical state line in the compression plane - specific volume ($v = 1 + e$ where e is void ratio) and mean stress - as the size ratio increases. This is an important first order effect on the mechanical response. The slope of the critical state line seems to fall slightly as the grading broadens: this might be considered a second order effect.

4 PARTICLE REMOVAL

The next series of tests were started as before. A sample with a given grading was compressed isotropically and sheared to a certain mobilised friction. The process of changing the grading by removal of particles was then performed as follows. From a state of equilibrium, the smallest disc in the sample was located and removed from the assembly by a sort of *deus ex machina*, with no attempt to describe any realistic erosion process. The removal of this particle leaves some unequilibrated interparticle forces in the assembly: external stresses were controlled and kept constant until deformations induced by these unequilibrated forces had converged (Fig 5a, b). This process of particle removal was then repeated. Two criteria have been defined for terminating this repeated process of particle removal: when the normal strain exceeds 25%; or when the size of the particle proposed for removal is equal to the 5% grain size (d_5) of the original sample. Under isotropic stresses, particle removal for all samples was limited by the latter condition; when particles were removed while the sample was subject to shear stresses, the limit criterion depended on the stress ratio as will be seen subsequently. These tests are actually testing a changing material for which the value of the ratio of maximum to minimum particle sizes R_D (or grading state index I_G [2]) is falling during the numerical test. If we plot results in any two dimensional diagram then we know that we are hiding some of the relevant information.

We have chosen to characterise the current state of the soil using the current value of the size ratio R_D so that the effect of the particle removal and the associated mechan-

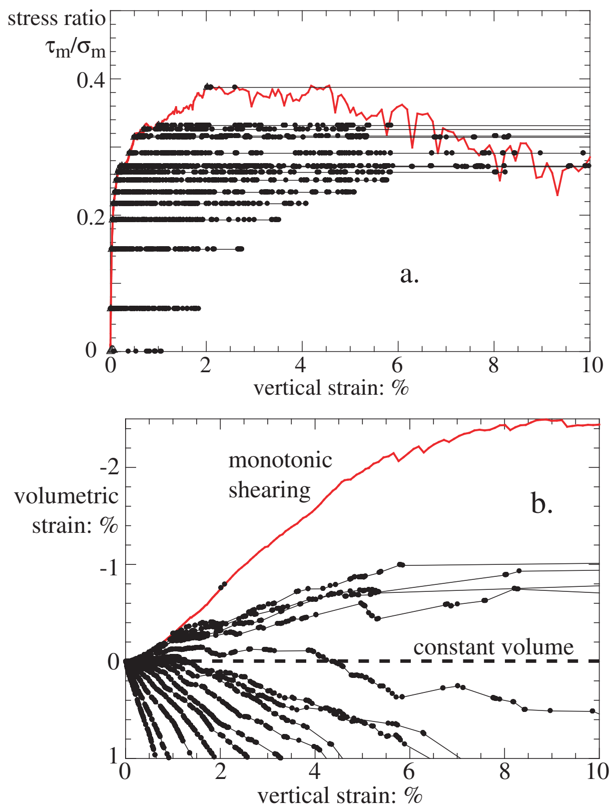

Fig. 5. Tests with particle removal ($R_D = 10$; $\sigma_m = 0.1$MPa): (a) stress ratio; (b) strain paths.

ical adjustment is to send the sample on a track across the specific volume:grading plane $v : R_D$ as both density and grading change together. It is perhaps slightly counterintuitional that the void ratio should increase in all these numerical tests even though the samples show some compressive volumetric strain. There are two competing effects: under constant external stresses the removal of soil particles destabilises the sample and it tends to compress. However, the effect of removing particles is to create a more open structure with higher void ratio - there is actually less material in the sample because of the removal of particles - and it is this effect that dominates.

Each process of particle removal starts from a different stress ratio within a standard monotonic test on material with constant grading. Just before the initiation of particle removal the sample is following a *strain* path which is broadly linked with the stress-dilatancy properties of the material: the rate of volume change is linked with stress ratio. The strain paths followed during particle removal (Fig 5b), with constant stress ratio, head off in a different direction. The test at

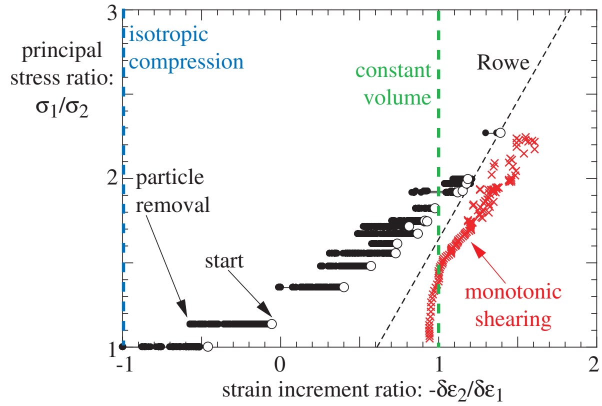

Fig. 6. Tests with particle removal ($R_D = 10$; $\sigma_m = 0.1$MPa): stress-dilatancy diagram.

stress ratio 0.291 deforms roughly at constant volume; all the paths for particle removal at lower stress ratio compress as they deform; all the tests with higher stress ratio expand. Figure 6 shows the strain increment ratios on a stress:dilatancy plot together with the continuous plot for a monotonic test and the flow rule proposed by Rowe [6]. Both the monotonic test and the particle removal tests at high stress level lie close to Rowe's line. The other tests start off the line and move further away as particle removal continues. The explanation of this behaviour is not yet clear. Since the imposed stresses are constant one supposes that the deformations that are occurring are purely plastic during particle removal though the early stages of the monotonic test may well be dominated by recoverable strains.

The two dimensional model used here is clearly a simplified analogue of the real three dimensional process of particle removal by internal seepage forces. Seepage will tend to remove particles with low contact constraint provided the void space is large enough for the particles to escape. Studies show that the coordination number tends to be larger for larger particles and that the constraint is less for smaller particles. We focus here on the deformation induced by the removal of one particle and the rearrangement that then controls the removal of the next particle. We have assumed that there is no constraint to the particle transport provided by the 3D void geometry. This provides a first stage towards the understanding of the mechanical macro-scale consequences of internal erosion.

5 STATE PARAMETER MODEL

On the evidence of tests with constant grading, we have proposed that there exist critical states for the material with eventual stress ratios which are somewhat independent of grading. Though the analyses show some not insubstantial fluctuation, the tests on dense samples with constant grading typically show a peak followed by softening to the critical state (Fig 2). Severn-Trent sand ([7], [8]) is a rather simple model for the mechanical behaviour of sand with unchanging grading which gives a central role to the critical state line through the state parameter ψ [9]. State parameter is the volumetric distance from the critical state line (Fig 7a). If we define the critical state line (locally) by

$$v_{cs} = \Gamma - \lambda \ln s \qquad (1)$$

then the current value of ψ is:

$$\psi = v - v_{cs} = v - \Gamma - \lambda \ln s \qquad (2)$$

We suggest that the strength of the soil is not a constant - except at the critical state - but is dependent on the density and stress level through the state parameter. Thus the *current* (peak) strength ρ_p is linked with the *current* value of ψ (Fig 7b):

$$\rho_p = \rho_c - k_1 \psi \qquad (3)$$

where ρ_c is the critical state stress ratio and k_1 is a soil constant. For loose material, with $\psi > 0$, the strength is below the critical state strength; for dense material, with $\psi < 0$, the strength is above the critical state strength. The terms 'dense' and 'loose' include effects of stress level s through the definition of ψ.

We have seen in Fig 4 that the dominant effect of changing grading is that the critical state line moves up (Γ increases) as the grading ratio falls (the change in slope λ is less significant). Parallel studies [3] suggest that it is the logarithm of the grading ratio - the geometry of the standard way in which the particle size distribution is presented (Fig 1) - that controls behaviour. So we write:

$$\Gamma = \Gamma_o - k_2 \ln R_D \qquad (4)$$

where Γ_o is the value of Γ for single sized material $R_D = 1$. In fact from Fig 4 we can estimate the constant $k_2 = 0.05/\ln 10 \approx 0.02$.

During particle removal at constant mean stress s the stress ratio ρ remains constant; the grading changes $\delta R_D > 0$ and the specific volume (or void ratio) increases $\delta v = \delta e > 0$ as

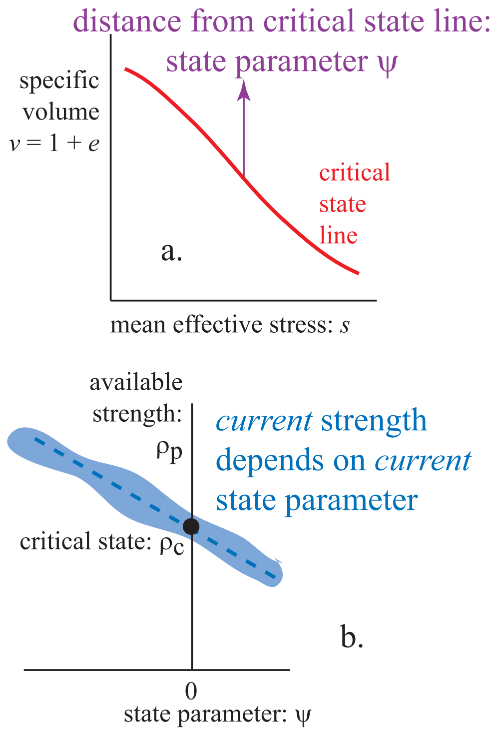

Fig. 7. (a) Compression plane, critical state line and definition of state parameter ψ; (b) link between *current* strength and *current* state parameter.

the sample responds to the particle removal. The change in state parameter $\delta\psi$ is the combination of these effects:

$$\delta\psi = \delta v - \delta\Gamma \qquad (5)$$

Particle removal tests which start with stress ratios greater than the critical state stress ratio may proceed stably at first but in general reach a state of continuing unstable deformation which is still continuing at the imposed 25% strain limit. This failure should be linked to the encounter by the sample with the softening part of the stress-strain response (Fig 2), where the current strength is no longer sufficient to sustain the chosen controlled constant stress ratio.

Severn-Trent sand assumes a hyperbolic hardening rule linking mobilisation of available strength with plastic distortional strain ϵ_s^p:

$$\frac{\rho}{\rho_p} = \frac{\epsilon_s^p}{\epsilon_s^p + b} \qquad (6)$$

where b is an indication of plastic stiffness: it is the strain required to mobilise 50% of the available strength. Equation (6) indicates that the available strength is never actually reached until infinite strain. The feedback mechanism provided by the stress:dilatancy relationship encourages the volumetric strain (compression or dilation) to move the soil towards the critical state as shearing proceeds so that when the peak strength is reached, at infinite strain, the state parameter has fallen to zero and the peak strength is in fact the critical state strength.

If we are removing particles at a stress ratio $\rho > \rho_{cs}$ then we may propose that failure will occur when the available strength has fallen to the presently demanded strength $\rho_p = \rho$ (strictly $\rho_p \approx \rho$ but the strain is large enough for the difference to be small). 'Failure' in this context means that we expect our numerical analysis to be unable any longer to converge. We know that stress controlled tests on softening material fail uncontrollably when a peak strength is reached (or in this case when the peak stress ratio reaches us).

We can generate an expression for a four dimensional strength hypersurface in terms of specific volume v, mean stress s, grading R_D and stress ratio ρ working from (3):

$$\begin{aligned}\rho &= \rho_{cs} - k_1(v - \Gamma_o + k_2 \ln R_D + \lambda \ln s) \\ &= [\rho_{cs} + k_1(\Gamma_o - \lambda \ln s)] - k_1[v + k_2 \ln R_D]\end{aligned} \tag{7}$$

We know $\rho_{cs} \approx 0.25$; with units of MPa for stress, $\Gamma \approx 1.22$ for $R_D = 2$ hence $\Gamma_o = 1.22 + 0.02 \ln 2 \approx 1.25$; $\lambda \approx 0.04/\ln 100 \approx 0.01$ from Fig 4. The stress level s is around 1MPa. The value of $k_1 \approx 1$ from tests on sands - perhaps the same value will apply to this two-dimensional material? So for a series of tests at constant mean stress $s = 1$MPa we have a three-dimensional failure surface:

$$\rho = 1.5 - v - 0.02 \ln R_D \tag{8}$$

and this criterion will govern the failure of removal tests with $\rho > \rho_{cs}$.

6 CONCLUSION

If the grading of a soil changes as a result of particle breakage or erosion then there is a change in the material which affects its mechanical behaviour. The mechanical response to the removal of particles by erosion can be studied against a framework which uses some measure of current grading as an additional state variable which influences strength just as importantly as density or stress level. The resulting multi-dimensionality is challenging in the presentation of results but throws light on the conditions under which erosion might lead to mechanical distress.

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