Experimental study of the generation of cyclic migration steps.

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Over a streambed, an interaction of flow and sediment results a bed modification, such as dune, ripple. In particular, due to an affection of Froude-supercritical flow over an erodible bed, a series of steps can occur. They migrate upstream while preserving its form. Here the features of these steps are considered by laboratory experiment.

The observation of the bed profile suggests that an unstable supercritical flow can cause the phenomenon. Moreover, the relations between the characteristic value (wavelength, wave height, and migration speed) and factors are investigated and the results show that dimensionless migration speed of step is proportional to the particle Reynolds number.

Key Words : *Bed shapes, Froude number, Flow instability, step migration speed, particle Reynolds number.*

1. INTRODUCTION

Over a steep slope channel such as mountain stream, characteristic bed shape such as step-pool bed forms can be created by flow and sediment transport. For example, upstream-migrating headcuts in the Greyfox channel have been reported by Reid (1989). Such bed shape modification plays an important role in sediment transport.

Parker and Izumi (2000) studied purely erosional cyclic steps that are similar to headcuts 1).

This research is motivated by the study by Taki and Parker (2005) 2). They observed cyclic steps in the laboratory and related their formation to Froude number.

This paper focuses on a process of bed modification due to the interaction of water flow and sediment transport. And the goal of this paper is to provide comprehensive results of cyclic steps.

2. EXPERIMENTAL METHOD

(1) Flume details

Experiments were carried out by using the flume shown in **Fig.1**. It was made of clear plastic, and has a length of 4m and a width of 2cm. The narrow width is designed to disregard the effect of cross section. A grid sheet was taped to the sidewall of the flume for measurement. At the downstream end of the flume, a low tail weir was set.

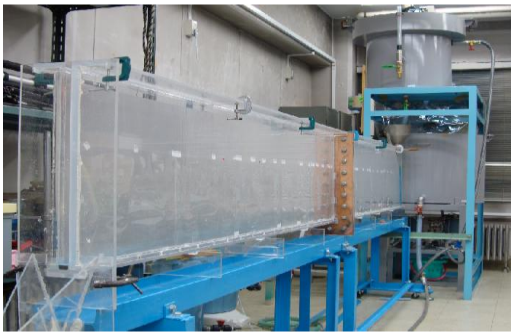

Fig.1 View of the experimental flume.

(2) Bed material

In this experiment, the two types of sediment were used. They were artificially sorted, commercially available sand. The respective grain sizes are 37 and 75μm. The sediment density ρ_s was close to 2.65.

(3) Experimental Procedure

Description of the experimental procedure is shown in **Fig.2**.

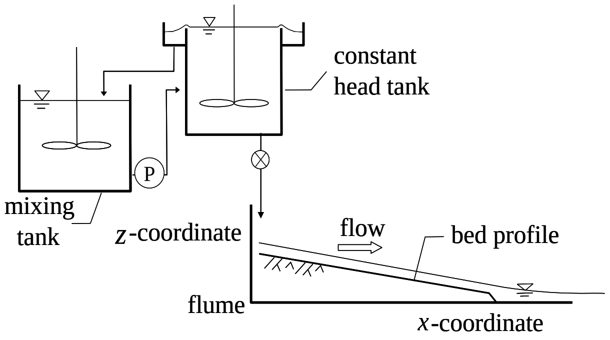

Fig.2 Experimental equipment.

Water and sediment were introduced from upstream end of the flume at constant rate. Water and sediment were mixed in the mixture tank before introduction into the flume. Due to the continuous feed of water and sediment, the bed aggraded, with keeping constant slope. Once the perturbation occurred over the bed, step formation began. The change of the bed state before and after step formation was regularly recorded by digital camera. The experimental data of flow depth were taken by measurement from the image.

3. EXPERIMENTAL RESULTS

Table 1 show the basic data obtained in the experiment. Here, d_{50} denotes median diameter of sediment, Q denotes Volume discharge of water and sediment, c denotes volume sediment concentration, h_n denotes flow depth, I denotes equilibrium slope, v_r denotes migration speed of steps, $\Delta\eta$ denotes step height, λ denotes step wave length, respectively. Definition of h_n , I , v_r , $\Delta\eta$ and λ is shown in **Fig.3**. Data of the case of 19 and 45μm is quoted from the experiment by Taki and Parker (2005) [2)].

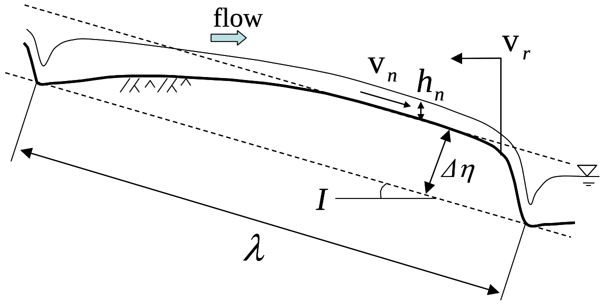

Fig.3 Definition sketch of variables.

Bed type in **Table 1** is classification of bed shape, two types are shown. The features of these are as follows.

- SS : single step. In this case, only a single step was observed.
- MS : Multiple cyclic steps. In this case, we observed more than two upstream-migrating steps.

The definition of these classifications is due to Taki and Parker(2005) [2)].

In table 1, h_n varies from 0.14 to 0.63cm. I varies from 0.017 to 0.118. v_r varies from 0.02 to 0.70cm/s. λ varies from 40 to 130cm. $\Delta\eta$ varies from 1.5 to 7.1cm.

4. PROCESS OF BED MODIFICATION

We traced the process of bed modification in the step formation. **Fig.4** shows the description of the bed state. The data used in Fig.4 is Run 2-10 (d_{50} =37μm, Q=16.9cm^3/s). Here, x is a streamwise coordination, η denotes bed height above a reference level, $\eta + h$ is a level of the water surface. The area where the data was taken is the range from x=56cm to x=156cm (at upstream end : x=0). Time t = 0 is the time of the beginning of introduction of water and sediment.

At t = 7202s, bedform never changed. Froude number is supercritical. At t = 14700s, bedform changes slightly. The shape of water surface changes with the change in the bed, and Froude number decreases in the downstream side. At t = 21005s, the step grows and the flow changes rapidly at a point of the hydraulic jump. At t = 33065s, it is seen that there are two flow zone in one wave length. That is, subcritical zone and supercritical zone. Because of these zones, erosion and deposition balance over one wave length. Moreover, flow is unstable in supercritical zone. This may cause the periodicity of steps.

5. ANALYSIS OF THE MIGRATION SPEED OF STEPS

Relation between sediment discharge in the supercritical zone Q_s and step migration speed v_r seems to be expressed as follows:

$$Q_s \Delta t = \mathrm{v}_r \, \Delta t \, \Delta\eta \, B \qquad (1)$$

where B is the flume width. Equation (1) is rewritten into the form

$$q_s = A_1 \, \mathrm{v}_r \, d_{50} \qquad (2)$$

Table 1 Experimental data.

Run	d_{50} [μm]	Q [cm^3/s]	c	Bed type	h_n [cm]	I	v_r [cm/s]	z [cm]	λ [cm]
1-1	19	78.8	0.00	SS	0.30	0.027	0.14	4.0	—
1-2	19	50.9	0.01	MS	0.27	0.040	0.21	3.2	130
1-3	19	37.5	0.02	MS	0.32	0.035	0.15	2.9	—
1-4	19	32.3	0.03	MS	0.20	0.047	0.10	3.8	—
1-5	19	26.3	0.04	SS	0.27	0.063	0.09	4.2	—
1-6	19	16.6	0.12	SS	0.22	0.091	0.11	5.7	—
1-7	19	11.7	0.18	SS	0.18	0.100	0.08	3.7	—
1-8	19	70.6	0.01	MS	0.38	0.037	0.11	3.9	75
1-9	19	26.1	0.07	SS	0.20	0.075	0.15	5.5	—
1-10	19	39.4	0.01	SS	0.22	0.047	0.10	3.9	—
1-11	19	25.4	0.04	SS	0.19	0.045	0.10	—	—
1-12	19	28.5	0.05	SS	0.20	0.045	0.16	4.1	—
1-13	19	36.0	0.04	MS	0.21	0.046	0.13	4.1	99
1-14	19	35.7	0.04	MS	0.30	0.061	0.14	2.6	64
1-15	19	35.9	0.04	MS	0.30	0.048	0.26	3.4	63
1-16	19	35.7	0.05	MS	0.27	0.054	0.15	5.0	57
1-17	19	35.6	0.06	MS	0.26	0.060	0.13	3.2	74
1-18	19	9.6	0.19	SS	0.17	0.094	0.07	2.0	—
1-19	19	17.5	0.11	SS	0.14	0.102	0.11	5.3	—
2-1	37	29.7	0.05	MS	0.63	0.034	0.19	7.1	126
2-2	37	11.7	0.01	MS	0.28	0.039	0.15	3.0	—
2-3	37	10.6	0.05	MS	0.32	0.076	0.22	4.1	125
2-4	37	29.4	0.05	MS	0.53	0.026	0.25	5.9	127
2-5	37	28.7	0.03	MS	0.43	0.017	0.25	4.6	100
2-6	37	17.2	0.03	MS	0.37	0.021	0.24	3.6	58
2-7	37	34.6	0.04	MS	0.68	0.017	0.24	4.5	113
2-8	37	25.7	0.02	MS	0.52	0.028	0.25	3.1	197
2-9	37	16.8	0.04	MS	0.37	0.059	0.28	3.4	89
2-10	37	16.9	0.02	MS	0.39	0.027	0.19	2.7	124
3-1	45	35.2	0.03	MS	0.19	0.046	0.24	2.4	95
3-2	45	35.2	0.06	MS	0.30	0.059	0.40	3.1	65
3-3	45	35.0	0.08	MS	0.22	0.051	0.33	3.0	79
3-4	45	41.9	0.03	MS	0.29	0.039	0.31	2.0	102
3-5	45	10.2	0.10	SS	0.23	0.118	0.06	1.9	—
3-6	45	20.6	0.05	MS	0.30	0.086	0.21	2.2	124
3-7	45	20.6	0.10	MS	0.20	0.088	0.27	3.0	120
4-1	75	17.0	0.01	MS	0.35	0.046	0.39	1.5	114
4-2	75	18.1	0.02	MS	0.45	0.068	0.49	1.8	42
4-3	75	16.4	0.04	MS	0.43	0.075	0.48	2.1	40
4-4	75	25.2	0.01	MS	0.61	0.059	0.54	2.3	51
4-5	75	21.2	0.02	MS	0.52	0.052	0.70	2.0	63
4-6	75	22.2	0.04	MS	0.53	0.063	0.52	1.8	56
4-7	75	25.1	0.05	MS	0.56	0.043	0.56	3.3	71

Notes : d_{50} = median diameter of sediment, Q = Volume discharge of water and sediment, c = volume sediment concentration, h_n = flow depth, I = equilibrium slope, v_r = migration speed of step, $\Delta\eta$ = step height, λ = step wave length, SS = single step, MS = multiple cyclic steps.

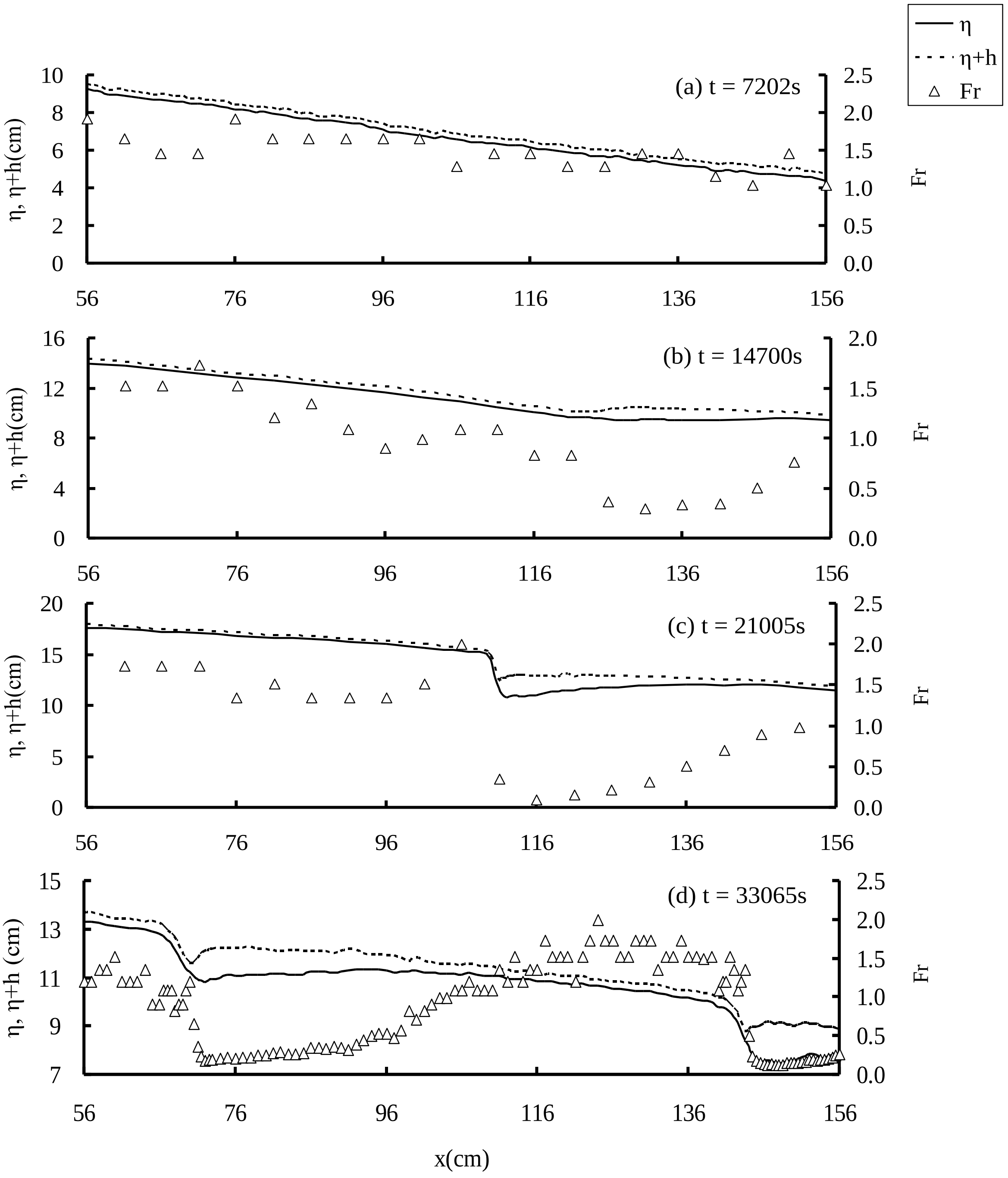

Fig.4 Diagram of bed height η , water surface level η +h and Fr. Flow is from left to right.

where $q_s = Q_s / B$, A_1 is coefficient.

Brown (1950) obtained a relation for dimensionless bedload transport rate in the following form :

$$\frac{q_s}{u_* d_{50}} = 10 \tau_*^2 \qquad (3)$$

where u_* is the shear velocity and τ_* is the dimensionless share stress.

Using equation (3), equation (2) reduce to

$$\frac{\mathrm{v}_r}{u_*} = \frac{10}{A_1} \tau_*^2 \qquad (4)$$

Relation between mean flow velocity v_n ($\mathrm{v}_n = \mathrm{Q}/Bh_n$) and u_* is obtained in the form

$$\frac{\mathrm{v}_n}{u_*} = \sqrt{\frac{8}{f}} \qquad (5)$$

where f is the friction coefficient.

Using equation (5), equation (4) reduce to

$$\frac{v_r}{v_n}=\frac{10}{A_1}\sqrt{\frac{f}{8}}\,\tau_*^{\,2} \qquad (6)$$

Since τ_* is the function of the particle Reynolds number Re_*, equation (6) seems to be rewritten as

$$\frac{v_r}{v_n}=\frac{10}{A_1}\sqrt{\frac{f}{8}}\,A_2\,\mathrm{Re}_*^{\,m} \qquad (7)$$

where A_2 and m are coefficient. Moreover, equation (7) reduce to

$$\frac{v_r}{v_n}=A_*\,\mathrm{Re}_*^{\,m} \qquad (8)$$

where

$$A_*=\frac{10}{A_1}\sqrt{\frac{f}{8}}\,A_2 \qquad (8a)$$

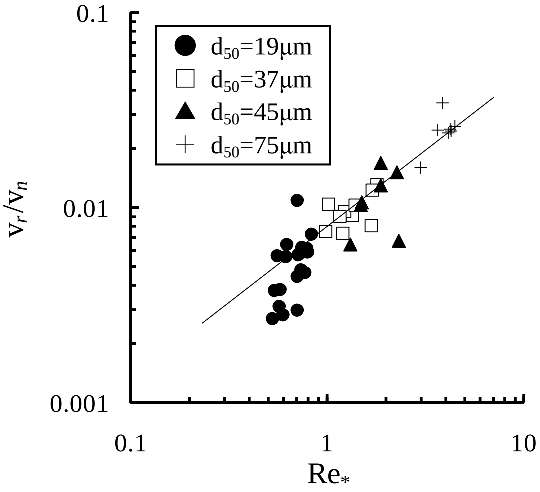

Fig.5 Plot of dimensionless step migration speed v_r/v_n versus particle Reynolds number Re*.

Fig.5 shows the plot of v_r/v_n versus Re_*. A regression analysis of the data yields the following equation

$$\frac{v_r}{v_n}=0.007\,\mathrm{Re}_*^{\,0.9} \qquad (9)$$

Re_* is given by the equation (10)

$$\mathrm{Re}_*=\frac{u_*d_{50}}{\nu} \qquad (10)$$

The value of u_* is the one at equilibrium slope and the value of kinematic viscosity of fluid ν is used the clear water value at $20\,^\circ\mathrm{C}$. Therefore, this relation suggests that the migration speed of the step strongly depends on particle Reynolds number.

6. CONCLUSION

The characteristic of cyclic steps is studied by laboratory experiment. Observation of the process of bed modification suggests that instability of flow may cause cyclic steps. Moreover, from the results of analysis of the migration speed of steps, it is found that step migration speed is proportional to the particle Reynolds number raised to the power of 0.9.

APPENDIX

A_1, A_2, A_* = coefficient
B = the flume width
c = volume concentration of sediment in feed
d_{50} = median diameter of sediment
f = the friction coefficient
F_r = Froude number
h_n = flow depth
I = equilibrium slope
m = coefficient
Q = Volume discharge of water and sediment
Q_s = sediment discharge in the supercritical zone
q_s =sediment discharge in the supercritical zone per unit width
Re_* = the particle Reynolds number
t = time
u_* = the shear velocity
v_n = mean flow velocity
v_r = migration speed of steps
x = streamwise coordinate
z = vertical coordinate
$\Delta\eta$ = step height
η = bed height above a reference level
$\eta+h$ = a level of the water surface
λ = step wave length
ν = kinematic viscosity of fluid
ρ_s = the sediment density
τ_* = the dimensionless share stress

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