FUZZY REGRESSION APPROACH TO ESTIMATING THE SETTLING VELOCITY OF SEDIMENT PARTICLES

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Fall velocity has a strong influence on river morphology, suspended sediment transport, and beach profile shape. In this research a fuzzy regression analysis is performed to study the fall velocity of non cohesive sediment particles (sand and gravel). Estimating the fall velocity of natural particles is related to some parameters. Especially, size of the particle plays an important role in the estimation of fall velocity. In addition, such parameter is difficult to precisely evaluate for natural particles and many uncertainties in measurement are existent. Deviations between the observed values and the estimated values and existent uncertainties are regarded as the fuzziness of the system's parameters and coefficients. A fuzzy regression model and definition of fall velocity as a fuzzy number might be very convenient and useful for finding a fuzzy structure in estimating the fall velocity of natural particles. In this paper, the fuzzy regression concept and its application in estimation of the fall velocity of natural particles by using experimental data are developed.

Keywords: *Settling Velocity, Sediment Particle, Fuzzy Regression, fuzzy number.*

1. INTRODUCTION

The settling velocity of sediment is one of the key variables in the study of sediment transport (Jimenez and madsen 2003). Hallermeier (1981) explains Settling velocity this way: ‘‘a sediment grain in a less dense, viscous fluid attains a terminal settling velocity as the gravitational force is balanced by the hydrodynamic drag force on the grain.’’ Fall velocity of a particle, depends on the density and viscosity of the fluid, and the density, size, shape, spherically, and the surface texture of the particle. Many attempts to predict the particle fall velocity have been carried out by researches, started by Stokes in 1851 [cited in Graf 1971] and followed by Oseen (1927), Rubby (1933), Rouse (1938), Interagency Committee (1957), Zanke (1977), Yallin (1977), Hallermier (1981), Dietrich (1982), Van Rijn (1989), Concharov [cited in Ibad-zadeh 1992], Julien (1995), Cheng (1997), Jimenez and Madsen (2003), Brown and Lawler (2003), She et al. (2005), and Wu and Wang (2006) among others, who developed empirical or semi-empirical relations for estimating the settling velocity of sediment particles.

Most of above mentioned investigations, however, have some limitations when it comes to applying them to engineering works. For instance, the relations developed by Stokes [cited in Graf 1971], Rouse (1938), Brown and Lawler (2003), are applicable only to spherical particles. Of course, it is well known that the shape of natural sediment particles departs from a sphere. This departure will have some consequences, one being that the settling velocity will be lower than that of a sphere with the nominal diameter. Due to the practical implications of this difference, several formulas have been proposed to calculate the settling velocity of natural sediments. All of these have been empirically derived, and in this sense they fit very well the data set employed. However, because new relationships have been proposed they have not incorporated previously published data to check the general accuracy of the respective formulas, and in this sense there is considerable uncertainty about which formula is the most accurate [cited in Jemenez and Madsen 2003]. Otherwise, an ancillary problem related to calculating the fall velocity is using the correct value of kinematic viscosity (Ahrens 2000). It can be seen that it is very inconvenient to make a decision on selecting an optimal fall velocity relation when several empirical or semi-empirical formulas give different answers to the same problem. In response to this uncertainty, in this research these deviations are regarded as the fuzziness of the system's parameters and coefficients. Thus, these deviations are reflected in a fuzzy definition of settling velocity and fuzzy regression model.

Lotfi Zadeh introduced the simple and intuitive concept of a fuzzy set in his paper in 1965 (Zadeh, 1965). Since then, fuzzy sets have been applied to a vast number of areas including environmental sciences: soil, forest and air pollution, meteorology, water resources, etc. (Bezdek, 1999). Fuzzy regression analysis was first proposed by Tanaka et al. (1980, 1982). They considered a regression model in which the relation of the variables is subject to fuzziness, i.e., the model with crisp input and fuzzy parameters.

In fuzzy regression, the relationship between the dependent and the independent variable is not as precise as the relationship in a conventional linear regression. In other words, fuzzy regression represents a phenomenon that is imprecise and vague by nature.

In traditional statistical inference, as we know, the regression models are used frequently in the researches of the relations among several variables in a system. Observing some of the variables, we can make estimates and predictions for the others. If a system under consideration is not governed by random variables and/or crisp observation but is governed by possibility variables and/or imprecise observation, it is more natural to seek a fuzzy regression analysis for such a system. Fuzzy regression, in a general way, can be classified into two categories:

I) Fuzzy regression when the relations of the variables are subject to fuzziness, II) Fuzzy regression when the variables themselves are fuzzy. It should be mentioned that, in some approaches more than one case is considered to provide a fuzzy regression model [cited in Taheri 2003].

In this paper, estimation of settling velocity is implemented by using fuzzy techniques rather than the usual and non-fuzzy techniques used in the past. There is a set of crisp observed data as input/output variables. Thus, the coefficient of fuzzy regression will be fuzzy and by using this model, it is possible to estimate fuzzy number of settling velocity for the related particle diameter.

The reasons are (1) estimation of size, shape and the surface texture of the particles and viscosity of the fluid is difficult to precisely evaluate in natural conditions; (2) the deviations of the data are suitably reflected in fuzzy parameters; (3) fuzzy regression is recommended for partially available data where human estimation is influential; (4) fuzzy regression model is useful in areas of decision making where there is a great deal of uncertainty as well as vague phenomena (Wen and Lee 1999) (5) the relation among parameters and effectiveness of them on each other and on estimated function are vague. Therefore, using fuzzy techniques tends to reduce the magnitude of errors and as a result, yields better forecasts and fitting for estimating the settling velocity of natural particles. One assumption of fuzzy regression is that the residual or deviation of the estimated value from the real value of the dependent variable is due to the fuzziness of the system's parameters (Wen and Lee 1999). This research could be treated as the first work to investigate fuzzy approach to estimating the settling velocity of the natural particles. The method of developing the new model is presented in the following sections. In the two next sections, basic theory of settling velocity and fuzzy regression are defined. After it, developing of fuzzy regression model for settling velocity and related results are explained.

1.1. SETTLING VELOCITY

In 1851, Stokes by using the Navier-Stokes equations along with a continuity equation expressed in polar coordinates, investigated the coefficient of drag applied by fluid flow upon a spherical particle (Graf 1971). Based on Stokes' results, the fall velocity of spherical particles in the region of particle Reynolds number (Re) less than 1, can be calculated using (Cheng 1997):

$$w = \frac{1}{18}\frac{g(s-1)d^2}{\upsilon} \qquad (1)$$

$$s = \frac{\rho_s}{\rho}$$

where:

w : particle fall velocity (m/s);

g : acceleration due to gravity (m/s2);

d : particle diameter (m);

υ : kinematic viscosity (m2/s);

s : relative density;

ρ_s : density of the sediment particle (kg/m3);

ρ : density of ambient fluid (kg/m3).

For natural sediment particles, many researchers have attempted to develop similar equations which are colleted in **Table 1**. Due to the extensive variation of natural particle geometry, however, there has been little success in this regard; therefore a large number of different equations, each of which can only be applied within a limited range of sediment and fluid conditions, are now available.

1.2. FUZZY REGRESSION

The fuzzy regression model can be considered as follow:

$$Y = \sum_{i=1}^{n} A_i x^n \qquad (7)$$

Where

Y : output fuzzy number;

x : input crisp variable;

n : the degree of proposed regression function that is equal to one at first;

A_i : coefficients of fuzzy regression model denoted as $A_i = (a_i, c_i)$ with center a_i and spread c_i.

Fig.1 illustrates details of nonlinear fuzzy regression and output fuzzy numbers. To estimate the fuzzy regression model, the required data set must be included input and output crisp variables $(x_j, Y(x_j))$. With these data and applying to fuzzy regression equations, fuzzy coefficient of it will be obtained. The output of regression is a fuzzy number which can be recognized by a fulfillment degree. This degree represents the fuzziness of output and varies between of zero and one (**Fig.1**). In this study, fulfillment degree is assumed zero for reaching maximum fuzziness of output. Therefore, the output value Y can be explained as a triangular fuzzy number $Y = (a, c)$ with center a and spread c.

2. DEVELOPING OF FUZZY REGRESSION MODEL FOR SETTLING VELOCITY

As mentioned in the past sections, it is always difficult to precisely evaluate the settling velocity and there are many uncertainties. Fuzzy set theory can be used for handling the existing uncertainties in estimating the settling velocity such as measurement errors, parameters which can't be calculated (e.g. particle shapes) and variety of effective parameters on the settling velocity.

Fuzzy regression model is developed to evaluate settling velocity by using experimental data set. The fuzzy regression is required to be fed with a number of x_j data and related $Y(x_j)$ data. Thus, to estimate the fuzzy regression model for settling velocity, the required data set includes particle diameter and settling velocity for corresponding particle. Four data sets, introduced by Jimenez and Madsen (2003), were used for validation and training Fuzzy model (**Table 2**). In **Table 2**, the number of data points from each source, n, Is listed in the third column. The data sets, first, were grouped into two groups. The first group, which was taken from Cheng (1997), Engelund and Hansen (1972), and Hallermeir (1981), corresponded to the settling velocities of natural sediments without an explicit definition of the shape factor, but taking it equal to 0.7, as it is usually taken as the most common value for naturally shaped sediments [for example, see Dietrich (1982)].

The Cheng (1997) data set is a compilation of Russian quartz sand experiments (original references can be found in Cheng's paper) in which the sediment size was characterized through the arithmetic average diameter (Cheng 1998). Because the specific gravity, s, was not given, it was assumed to be 2.65.

The Engelund and Hansen (1972) data set was taken from Fredsoe and Deigaard (1992). The sediment size was characterized through the sieve diameter ds (not used here), and the nominal diameter dN. Similar to Cheng's 1997 data set, the specific gravity s was not given, so it was assumed to be 2.65.

The Hallermeir (1981) data set is a compilation of previously published experiments (original refrences can be found in Hallermeier's paper), in which the sediment size was characterized by the sieve diameter.

Since the method proposed here was derived to be used with the nominal diameter, the given sieve diameters were, as previously mentioned, converted to nominal diameter by using the rule of

Table 1 List of relations presented for estimating Settling velocity

Originator	Main relation	Comments
Zanke (1977)	$w=\frac{10\upsilon}{d}\left[\left(1+0.01\frac{(s-1)g\,d^3}{\upsilon^2}\right)^{0.5}-1\right]$	
Hallermier (1981)	$Re=\frac{D_{gr}^3}{18}$ $Re=\frac{D_{gr}^{2.1}}{6}$ $Re=1.05D_{gr}^{1.5}$	$D_{gr}<3.42$ $3.42<D_{gr}<21.54$ $D_{gr}>21.54$
Van Rijn (1989)	$w=\frac{1}{18}(s-1)\frac{gd^2}{\upsilon}$ $w=1.1\sqrt{\left[(s-1)g\,d\right]}$ $w=10\frac{\upsilon}{d}\left[\sqrt{(1+0.01\,d^3)}\right]$	$d<0.01\,cm$ $d>0.1\,cm$ $d=0.1\,cm$
Concharov (1962)	$w=\frac{1}{24}\frac{g(s-1)d^2}{\nu}$ $w=1.068\sqrt{(s-1)\,gd}$	$d<0.015\ cm$ $d>0.15\ cm$
Zhang (1993)	$w=\sqrt{\left[(13.95\frac{\upsilon}{d})^2+1.09(s-1)\,gd\right]}-13.95\frac{\upsilon}{d}$	
Zhu & Cheng (1993)	$w=\frac{\upsilon}{d(9\cos^3\alpha+1.8\sin^2\alpha)}[-24\cos^3\alpha+$ $(576\cos^6\alpha+(18\cos^3\alpha+3.6\sin^2\alpha)D_{gr}^3)^{0.5}]$	
Soulsby (1997)	$w=\frac{10.36\,\upsilon}{d}\left[(1+0.156\frac{(s-1)g\,d^3}{16\,\upsilon^2})^{0.5}-1\right]$	
Cheng (1997)	$\frac{wd}{\upsilon}=\left(\sqrt{25+1.2d_*^2}-5\right)^{1.5}$ $d_*=\left(\frac{(s-1)g}{\upsilon^2}\right)^{0.333}d$	$csf=0.7$
Jimenez and Madsen (2003)	$\frac{w}{\sqrt{(s-1)g\,d}}=\frac{1}{A+\frac{B}{S_*}}$ $S_*=\frac{d}{4\upsilon}\sqrt{(s-1)g\,d}$	$csf=0.7$
She et al. (2005)	$R_e=1.05D_{gr}^{1.5}\left[1-exp\left(-0.08D_{gr}^{1.2}\right)\right]$ $R_e=1.05{D_{gr}}^{1.5}\left[1-exp\left(-0.315D_{gr}^{0.765}\right)\right]^{2.2}$	$D_{gr}>2$ $D_{gr}<2$
Wu and Wang (2006)	$w_s=\frac{M\nu}{Nd}\left[\sqrt{\frac{1}{4}+\left(\frac{4N}{3M^2}{D_*}^3\right)^{1/n}}-\frac{1}{2}\right]^n$	$M=53.5e^{-0.65S_f}$ $N=5.65e^{-2.5S_f}$ $n=0.7+0.9S_f$

Table 2 Data used for training Fuzzy model. [Cited in Jimenez and Madsen 2003]

			d (mm)		w (m/s)	
Code	Data	n	Range	Mean	Range	Mean
1	Cheng (1997)[a,c]	37	0.061-4.5	1.15	0.002-0.281	0.101
2	Engelund and Hansen (1972)[c,d]	22	0.100-1.9	0.580	0.005-0.170	0.063
3	Hallermeier (1981)[b,c,d]	15	0.152-0.61	0.369	0.017-0.075	0.045
4	Raudkivi (1990)[d,e]	12	0.200-2.0	0.930	0.018-0.194	0.098

[a]Arithmetic average diameter (d_n)

[b]Assuming $d_s/d_N \approx 0.9$

[c]Shape factor not given, assumed 0.7

[d]Nominal diameter (d_N)

[e]Shape factor=0.7

In all data set: Sg=2.65

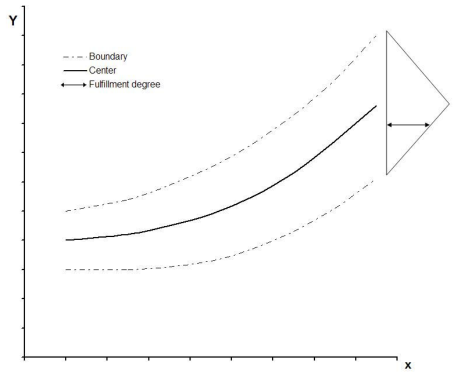

Fig.1 Fuzzy regression components and fuzzy output number.

thumb ds/dN ≈ 0.9 (Raudkivi 1990). As an example of the applicability of this approach, the ratio calculated from the data supplied by Engelund and Hansen (1972) gives a mean value of 0.93 with a standard deviation of 0.04. The analysis in this research is restricted to sands having size in the quartz range. Hence, only experiments with a specific gravity equal to 2.65 between 2.57 and 2.67 were considered. The original compilation of Hallermeier's data set also included the Engelund and Hansen (1972) data, but it was considered in this research separately.

The second group of data sets corresponded to the sediment settling velocities reported by Raudkivi (1990), originally given by the U.S. Inter-Agency Committee. The reported data consist of settling velocities of sediment characterized by its nominal diameter and shape factor. Among those, shape factor equal to 0.7 were used here (**Table 2**).

With the first try, it was obviously resulted that the fuzzy regression must be fitted on data set in two parts. The best boundary of different parts of fuzzy regression is obtained on $d = 0.001\,m$ which is proposed by Van Rijn (1989). Therefore, with available data and applying nonlinear fuzzy regression equations, the fuzzy regression model of settling velocity was obtained. The best degree of proposed regression models is equal to two. A_0 and A_1 are triangular fuzzy numbers. **Table 3** illustrates characteristics of fuzzy nonlinear regression models and the fuzzy regression curves, and observed crisp data are shown in **Fig.2.** By using the proposed fuzzy regression model, it is possible to estimate the fuzzy triangular value of settling velocity for corresponding particle diameter. For example, for particle equal to 0.0005 m, the calculated settling velocity is equal to (0.0605,0.0065). This triangular fuzzy number is shown in **Fig.3.**

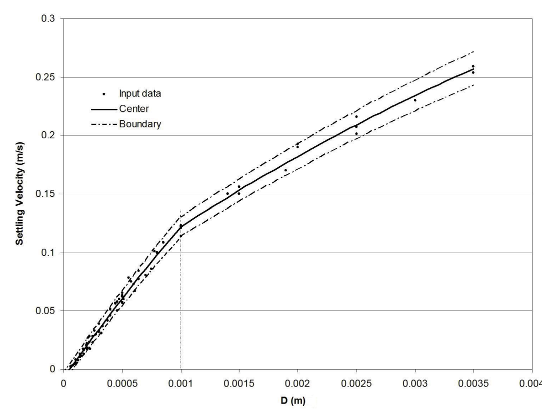

Fig.2 Nonlinear fuzzy regression curves for estimating the settling velocity of sediment particles.

In order to evaluate the developed models, settling velocity is calculated for some particle diameter by using 11 methods of other researchers collected in **Table 1** and the results are shown in **Fig.4.** As shown in **Fig.4**, these methods estimate various settling velocity for one particle diameter. It means that arrangement of settling velocities, calculated by various methods for different particle diameters is not constant. In other words, when a method estimates settling velocity for a sediment particle, greater than other methods, it is possible that for other sediment particle it doesn't estimate the largest value of settling velocity. Finally, it can't be said which method estimates settling velocity exactly. Thus, Fuzzy definition of settling velocity is the best solution of estimation it by using particle diameter. The fuzzy regression shown in **Fig.4**, covers the most of other methods. Although proposed method in this paper defines a novel definition of settling velocity, it accepts all of past methods.

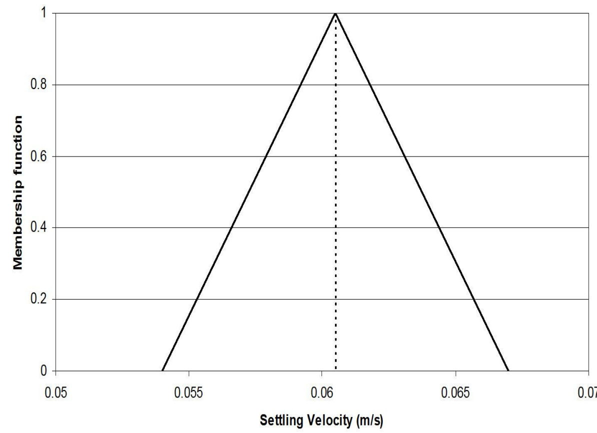

Fig.3 Sample of triangular fuzzy value of settling velocity in d=0.0005(m)

Table 3 Characteristic of fuzzy regression model for estimation of settling velocity

Limit of Diameter	A_0		A_1		A_2	
	a_0	c_0	a_1	c_1	a_2	c_2
$D < 0.001m$	-0.007	0.004	141	5	-12000	0
$D \geq 0.001m$	0.054	0.0057	72	2.5	-4000	0

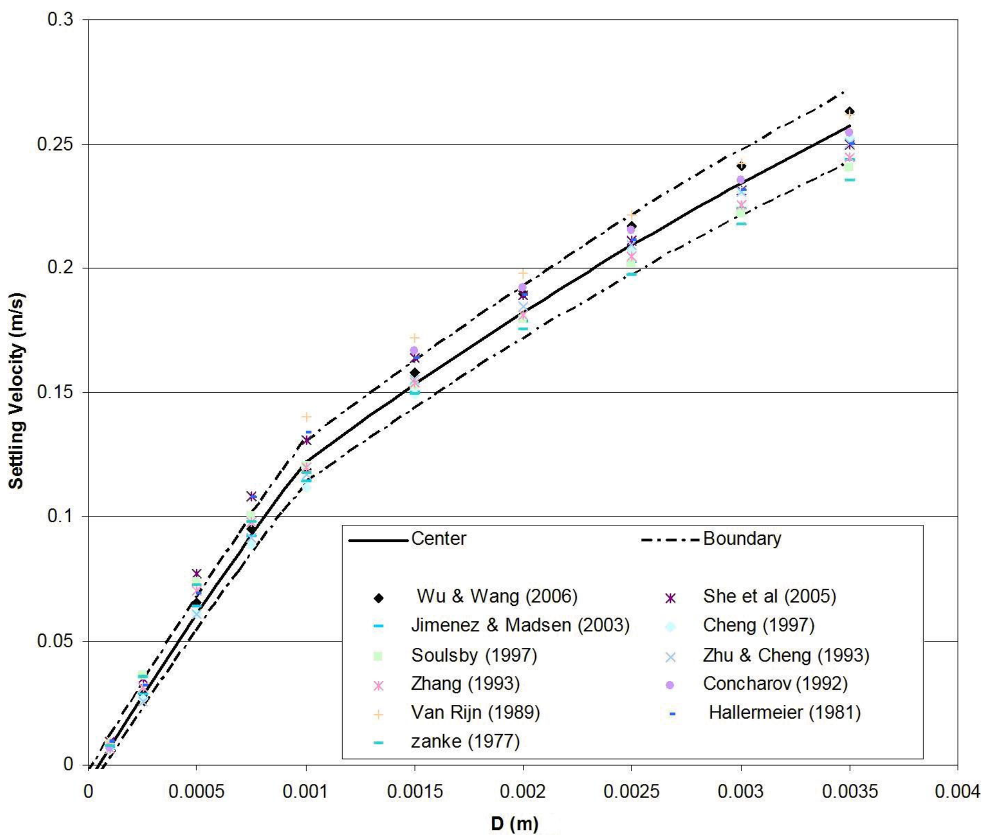

Fig.4 Estimated settling velocity by using other researchers' methods and fuzzy regression curves

3. SUMMARY AND CONCLUSIONS

A nonlinear Fuzzy model was developed for evaluating the settling velocity of non cohesive sediment particles (sand and gravel). With the use of the 86 sets of experimental data and results, fuzzy regression approach was introduced to develop the process models for settling velocity. This model estimates the triangular fuzzy number of settling velocity for each sediment diameter.

Comparison with past published methods shows that the proposed model has an innovation in definition of settling velocity with fuzzy set theory and estimation by using fuzzy regression model.

Although the proposed model applies past methods by using a fuzzy definition, uncertainties are incorporated in prediction of settling velocity.

Future research will be concerned with applying fuzzy set theory to estimation of other parameters in sediment studies and to developing sediment model. Since, there is more uncertainties in sediment issues, fuzzy set theory is very useful method for incorporating them into the models.

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